NICKS & TRICKS

LUKE'S GUIDE TO LEAVING CERT HL MATHS

Topic 1 – Algebra

Algebra is the MOST vital part of your entire Leaving Cert Maths Exam! Around 30% of questions on your exam will contain entirely algebra sections and algebra ideas will be used in every single area of maths! Learn the below nicks & tricks to help you find "x":

(i) Revision of Junior Cert Algebra
(ii) Factorising
(iii) Simultaneous Equations
(iv) Algebraic Identities
(v) Cubic Equations
(vi) Inequalities
(vii) Surds
(viii) Indices
(ix) Logs

(i) REVISION OF JUNIOR CERT ALGEBRA

(A) Algebraic Fractions

2 Different Types

+ or −

$$\frac{2}{3} + \frac{x}{4}$$

UP, UP, BOTTOM BOYS

$$\frac{(2)(4) + (3)(x)}{(3)(4)}$$

$$\frac{8 + 3x}{12} \quad \checkmark$$

=

$$\frac{2}{3} = \frac{x}{4}$$

UP, UP, NO BOTTOM BOYS

$$(2)(4) = (3)(x)$$

$$8 = 3x \quad \checkmark$$

(B) Simultaneous Equations

1. Label both equations

2. Multiply one or both equations so that the x's or y's are equal and opposite and then cancel!

Don't forget to sub your answer for x or y back in to get the other one!

$$5x + 4y = 37 \quad (1)$$
$$x + 2y = 11 \quad (2)$$

$$5x + 4y = 37 \quad (1)$$
$$-2x - 4y = -22 \quad (2) \times -2$$
$$3x = 15$$

(See section (iii) Simultaneous Equations for the new types that come up for LC!)

(C) Algebraic Long Division

The key to these questions is that you're only ever dividing by the number with the x in it! Never divide by the other number!

1. Divide by number with x in it.
2. Multiply answer by both numbers you are dividing by and add these underneath.
3. Swap sign for these numbers and repeat process.

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
\hline
2x + 1 & 2x^3 + 11x^2 + 17x + 6 \\
 & \underline{-2x^3 - x^2} \\
 & \qquad 10x^2 + 17x + 6 \\
 & \qquad \underline{-10x^2 - 5x} \\
 & \qquad\qquad 12x + 6 \\
 & \qquad\qquad \underline{-12x - 6} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

Make sure you get a 0 at the end! If you don't you have gone wrong somewhere!

(D) Factorising

4 Different Types

Highest Common Factor

$8x + 12x^2$

$4x(2 + 3x)$

1. Divide by biggest thing that goes into both terms.
2. Write it outside a bracket!

Factorise by Grouping

$5fh - 2h^2 + 15f - 6h$

$h(5f - 2h) + 3(5f - 2h)$

$(h + 3)(5f - 2h)$

1. Do 2 Highest Common Factors.
2. Combine things outside brackets into their own bracket.
3. Write down the repeating bracket once!

If your brackets don't match, try moving around some terms before you start!

Difference of 2 Squares

$16a^2 - 64$

$(4a - 8)(4a + 8)$

1. Write down:

(+)(-)

2. Put square root of the term on the left into left side of both brackets.
3. Square root of right term into right side of both brackets (ignore -)

Quadratics

$2x^2 + 8x + 8$

$2 \times 8 = 16$

$4 + 4 = 8$

$4 \times 4 = 16$

$2x^2 + 4x + 4x + 8$

$2x(x + 2) + 4(x + 2)$

$(2x + 4)(x + 2)$

1. Multiply first and last number (ignore -).
2. Factors of this number that give you middle number?
3. Swap out middle number for these 2 numbers.
4. Do Factorising by Grouping!

(ii) FACTORISING

In addition to the 4 types of factorising you learned in Junior Cert, there are an extra 2 types of factorising you'll need for Leaving Cert.

SUM OF 2 CUBES

$$(x + y)(x^2 - xy + y^2)$$

NOT in Log Tables! Must Learn off!

$$64a^3 + 125b^3$$

$$(4a + 5b)(16a^2 - 20ab + 25b^2)$$

1) Write down formula.
2) Cube root of left term is x and cube root of right term is y.
3) Fill in formula.

DIFFERENCE OF 2 CUBES

$$(x - y)(x^2 + xy + y^2)$$

NOT in Log Tables! Must Learn off!

$$64a^3 - 125b^3$$

$$(4a - 5b)(16a^2 + 20ab + 25b^2)$$

1) Write down formula.
2) Cube root of left term is x and cube root of right term is y.
3) Fill in formula.

(iii) SIMULTANEOUS EQUATIONS

2 Different Types

LINEAR

$$2x + y + z = 8$$
$$5x - 3y + 2z = 3$$
$$7x + y + 3z = 20$$

Same as JC simultaneous equations just longer!

1) Label equations.
2) Multiply 1 or 2 equations to make numbers in front of x, y or z equal and opposite.
3) Do this again so you have 2 new equations with just 2 unknowns in each.
4) Solve like in JC.
5) Sub answers back in to get other unknowns.

NON-LINEAR

$$x + y = 1$$
$$x^2 + y^2 = 25$$

One equation has a x^2 and one just has x!

1) Change x equation into x = ... or y = ...
2) Sub this into x^2 equation.
3) Solve quadratic.
4) Sub values back into x equation to get other unknown.

(iv) ALGEBRAIC IDENTITIES

These are where you match x^2 on the left-hand side of = with x^2 on the right, x on the left with x on the right etc.

Example: Solve for a & b

$$4x^2 + 4ax + a^2 = 4x^2 + 12x + b$$

x^2	x	*Numbers without x*
$4x^2 = 4x^2$	$4ax = 12x$	$a^2 = b$
	$4a = 12$	$3^2 = b$
	$a = 3$	$b = 9$

The key to these is just matching! Don't overcomplicate it!

(v) CUBIC EQUATIONS

These are equations with x^3 in them. They have 3 values for x that you need to find. There is a 4 step process to these:

1) Find the 1st x value by trial and error.
2) Factor Theorem.
3) Divide factor into cubic equation.
4) Solve quadratic for other 2 x values.

Example:

Solve $x^3 - 2x^2 - 11x + 12 = 0$

1) Using a calculator, sub numbers in for x and find out what equals 0.
   Try 1, -1, 2, -2, 3 and -3. It will be one of those!
   $(1)^3 - 2(1)^2 - 11(1) + 12 = 0$
   $\rightarrow x = 1$

2) Use the Factor Theorem. If $x = 2$ worked, then $x - 2$ is your factor. If $x = -3$ worked, $x + 3$ is your factor etc.
   $x = 1 \rightarrow x - 1$ is factor

3) Divide this factor into cubic equation using algebraic long division.

$$
\begin{array}{r|rrrr}
 & x^2 & - x & - 12 & \\
\cline{2-5}
x - 1 & x^3 & - 2x^2 & - 11x & + 12 \\
 & \underline{-x^3} & \underline{+ x^2} & & \\
 & & -x^2 & - 11x & + 12 \\
 & & \underline{x^2} & \underline{- x} & \\
 & & & -12x & + 12 \\
 & & & \underline{12x} & \underline{- 12} \\
 & & & & 0
\end{array}
$$

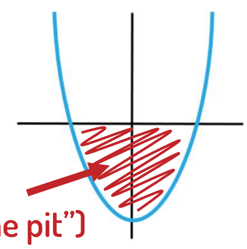
4) Solve the quadratic that comes out to get your other 2 x values.
   $x^2 - x - 12 = 0$
   $(x + 3)(x - 4) = 0$
   $x = -3, \quad x = 4$

(vi) INEQUALITIES

Inequalities are equations that use $<, \leq, >$ or $\geq$ instead of $=$.

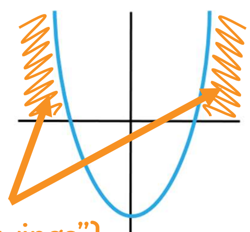
For all inequalities, just treat the $<, \leq, >$ or $\geq$ sign as a $=$ and solve normally! There are just a few things that are different:

1) If you change the signs of every number in the equation, you need to swap the direction of the symbol!
   Example:
   $-x < 6$
   $x > -6$
2) Graph your answers when you're finished! (Look at sign next to 0 before solving the quadratic)



(Think "the pit")

If ≤ 0
(The crocodile is going to eat the 0 so prepare the pit!)



(Think "wings")

If ≥ 0
(The crocodile isn't eating the 0 so it flies away!)

3) Present your answer using ≤ and ≥. (Look at your graph to help!)

There are 3 different types of inequalities they will ask you which have different ways of starting them.

3 Different Types

Quadratic

$$2x^2 + 7x + 6 \geq 0$$

Solve as a quadratic as normal.

Modulus

$$|x - 4| \geq 2$$

$$(x - 4)^2 \geq (2)^2$$

$$x^2 - 8x + 16 \geq 4$$

Square both sides to get rid of the | |.

Fractional

$$\frac{2x - 3}{x + 2} \geq 3$$

$$(x + 2)^2\left(\frac{2x - 3}{x + 2}\right) \geq 3(x + 2)^2$$

Multiply both sides by both denominators squared.

(vii) SURDS

Surds are numbers like $\sqrt{2}$, $\sqrt[3]{4}$, $\frac{\sqrt{3}}{2}$. In an exam, they'll want surd answers in simplest form so if you're ever unsure, put a surd into a calculator to get it in simplest form.

In an equation with 1 surd in it, make sure the surd is on its own on one side of the = and then square both sides!

$$x = \sqrt{5x - 4}$$

$$(x)^2 = (\sqrt{5x - 4})^2$$

$$x^2 = 5x - 4$$

In an equation with 2 surds in it, put one surd on each side of the = and then square both sides!

$$\sqrt{2x + 7} = 2 + \sqrt{x}$$

$$(\sqrt{2x + 7})^2 = (2 + \sqrt{x})^2$$

$$2x + 7 = 4 + 4\sqrt{x} + x$$

N.B. When you have gotten your answers for a surd equation, you must check your answers by subbing them back into the equation to see if they work! If one answer does work and one doesn't, just draw an "x" next to the answer that doesn't work.

(viii) INDICES

"Indices" just means questions with powers e.g. $2^{2x-1} = 8^{3x-5}$. With these questions, we need to remember a few rules:

1. Multiplying = Add the Powers → $x^3 \times x^2 \times x^4 = x^9$
2. Dividing = Subtract the Powers → $\frac{x^5}{x^3} = x^2$
3. Brackets = Multiply the Powers → $\left(x^{3y}\right)^2 = x^{6y}$
4. Anything to the power of 0 = 1 → $x^0 = 1$
5. To bring above or below the line, change the sign → $x^{-2} = \frac{1}{x^2}$
6. Square root is same as to power of a half → $\sqrt{x} = x^{\frac{1}{2}}$

With these questions, you will need to be able to use these 6 rules to move things around or simplify terms. The method for doing these questions is mostly the same:

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

The METHOD to most of these questions is to rewrite everything in the question as the same number to different powers!

$$2^{2x-1} = 8^{3x-5}$$

Rewrite 8 as 2^3 (As our method tells us to)

$$2^{2x-1} = (2^3)^{3x-5}$$

Multiply out bracket using Rule #3

$$2^{2x-1} = 2^{9x-15}$$

Put powers equal to each other

$$2x - 1 = 9x - 15$$

$$7x = 14$$

$$x = 2$$

Page 21 also has our 6 rules on the left-hand side!

(ix) LOGS

Séana agus logartaim		Indices and logarithms
$a^p a^q = a^{p+q}$	$\log_a(xy) = \log_a x + \log_a y$ ★	$a^x = y \Leftrightarrow \log_a y = x$
$\frac{a^p}{a^q} = a^{p-q}$	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ ★	$\log_a(a^x) = x$
$(a^p)^q = a^{pq}$	$\log_a(x^q) = q\log_a x$ ★	$a^{\log_a x} = x$
$a^0 = 1$	$\log_a 1 = 0$	
$a^{-p} = \frac{1}{a^p}$	$\log_a\left(\frac{1}{x}\right) = -\log_a x$	$\log_b x = \frac{\log_a x}{\log_a b}$
$a^{\frac{1}{q}} = \sqrt[q]{a}$		

For this section, page 21 of the Log Tables is your key!

Especially the equations with stars next to them here.

$\boxed{Log_2 8}$ → Means "The number we put 2 to the power of to get 8" i.e 3

Knowing this, we can work out how to get rid of a Log in question e.g:

$$Log_2 8 = x$$
$$2^x = 8$$
$$2^x = 2^3$$
$$x = 3$$

We put the base number in the Log to the power of what's on the other side of the equals, and we bring the big number in the Log over to the other side!

$$Log_x 25 = 2$$
$$x^2 = 25$$
$$x = 5$$

BREAKTHROUGH MATHS

For any question with logs, just look at page 21 of the Log Tables (especially the 3 equations I have put stars next to) and try to manipulate the question in whatever way you can!

$$\text{Given } Log_a 2 = p \ \& \ Log_a 3 = q:$$
$$\text{Find } Log_a \frac{8}{3} \text{ in terms of } p \ \& \ q$$

$$Log_a \frac{8}{3}$$

Page 21 middle column, 1st equation

$$Log_a 8 + Log_a 3$$

8 is the same thing as 2^3

$$Log_a 2^3 + Log_a 3$$

Page 21 middle column, 3rd equation

$$3Log_a 2 + Log_a 3$$
$$3p + q$$

Logarithmic Functions

These are any functions that use "e" or "ln". These questions are the same as functions questions, but they have one thing that always comes up that you need to remember!

Whenever you are trying to solve for something that is inside the power of e:

1) Get the ln of both sides.
2) Now you are allowed to take the power outside the ln.
3) ln(e) = 1 so we can get rid of it.
4) Solve rest as usual! (You can get what ln(20) is by putting it into your calculator).

$$e^{10x} = 20$$
$$ln(e^{10x}) = ln(20)$$
$$10xln(e) = ln(20)$$
$$10x = ln(20)$$
$$x = \frac{ln(20)}{10}$$
$$x = 0.3$$

LUKE'S EXAM PREDICTIONS

- Factorising & Junior Cert Algebra are needed <u>every</u> year!
- Simultaneous Equations have come up 4 out of the past 7 years!
- Logarithmic Functions have come up 4 out of the past 7 years!
- Cubic Equations have come up 3 out of the past 7 years!
- Inequalities have come up 3 out of the past 7 years!
- Logs have come up 3 out of the past 7 years!
- Algebraic Identities has come up 2 out of the past 7 years!
- Surds have come up 2 out of the past 7 years!

<u>NOTE:</u> Algebra is the most important topic and so even if a section doesn't come up one year, it's key ideas may be used in a different type of question and so it is best to be comfortable even with algebraic topics that might not come up as often as others!

If you study this guide, you'll have a strong grasp of the most important part of your Leaving Cert Maths Course! Understand how to tackle and start every type of question laid out here and you will notice a huge improvement in your mathematical ability!

"The maths exam is about progress, not perfection!"