



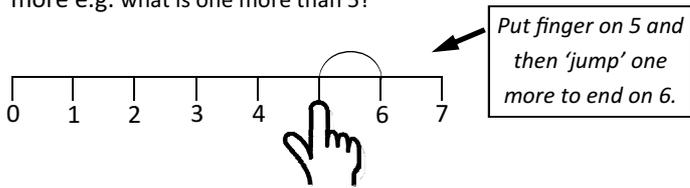
Addition, +

Addition begins on the simplest level using **objects** to see totals as **one more** and **two more** are added. Children also investigate different ways to make a total.

The focus at this level is 'seeing' the addition taking place and becoming familiar with some mathematical vocabulary e.g. 1 more, 2 more, add 2 more, add 1 more, altogether.

The written calculation (number sentence) may be shown to the children ($4 + 1 = 5$) but they would not be expected to write it themselves.

A number line might be used to 'jump' on one more or two more e.g. what is one more than 5?



Later, children will be able to add larger numbers using objects, but the corresponding calculation will be written alongside.

Children will be encouraged to 'jump' up or forward on the number line to find the new total, up to 20. They will recognise that addition can be done in any order ($6+4$ or $4+6$) but will usually begin with the largest number to make counting on easier.

Gradually children are drawn away from the practical (using objects) and closer to written addition.

The number line will still be used to add on larger numbers (usually below 20) but the children will be able to record answers as a written calculation (e.g. $13+6=19$) and may add more than 2 numbers.

The next stage of written addition is 'partitioning' - into 'tens and units', then 'hundreds, tens and units'.

Partitioning encourages children to know the true value of numbers; e.g.:-

35 can be partitioned into 30 and 5

35 + 23

$30 + 20 = 50$

$5 + 3 = 8$

$50 + 8 = 58$

So...**35 + 23 = 58**

Notice that the tens are added together first, then the units.

The totals can then be added mentally or as a written calculation. Children may circle the totals that they need to add.

Partitioning can also be applied to 3-digit numbers; e.g.:-

325 + 456

$300 + 400 = 700$

$20 + 50 = 70$

$5 + 6 = 11$

So...**700 + 70 + 11 = 781**

To move children to a standard written method, the style of presentation changes to:-

$$\begin{array}{r}
 35 \\
 + 23 \\
 \hline
 50 \quad (30 + 20) \\
 \underline{\quad 8} \quad (5 + 3) \\
 58
 \end{array}$$

Notice how the high value for each number is added first.

Keeping digits closely in columns becomes important here.

This strategy can be applied to 3-digit numbers

$$\begin{array}{r}
 524 \\
 + 423 \\
 \hline
 900 \quad (500 + 400) \\
 \quad 40 \quad (20 + 20) \\
 \underline{\quad 7} \quad (4 + 3) \\
 947
 \end{array}$$

As children's maths understanding develops they will realise that addition can be done in any order.

524		524
$+ 423$		$+ 423$
900	...this is the same as...	7
40		40
$\underline{7}$		$\underline{900}$
947		947

If children are ready, they will finally be introduced to 'the compact method' of addition.

See the methods supplement for an explanation of how this method is used.

625		645
$+ 48$...or...	$+ 78$
$\underline{673}$		$\underline{723}$
1		1 1



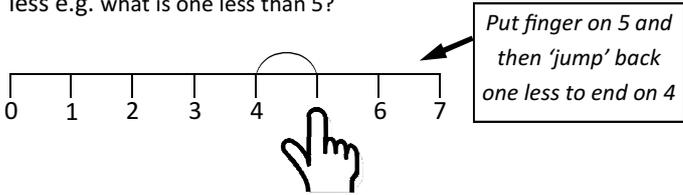
Subtraction, -

Subtraction begins similarly to addition using **objects** to 'take one less' and 'two less'.

Once again the focus is on 'seeing' the subtraction taking place.

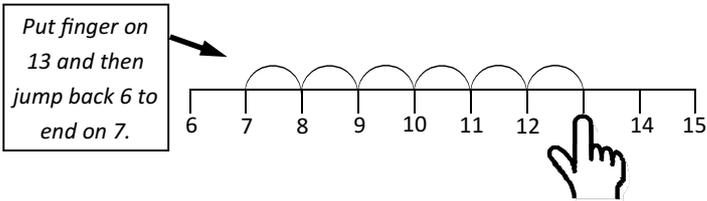
The written calculation (number sentence) may be shown to the children ($5 - 1 = 4$) but they would not be expected to write it themselves.

A number line might be used to 'jump' back one less or two less e.g. what is one less than 5?



Children will then progress to subtract larger numbers, still using objects to visualise the process, with the written calculation alongside (e.g. $13 - 6 = 7$).

Using number lines, children will be asked to 'find the difference' between two numbers e.g. what is the difference between 13 and 6?

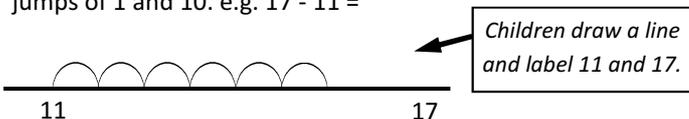


The method of counting back on a number line will be continued until children are confident with the process of subtraction and are ready to move onto larger numbers, typically above 20.

More complex language will now be used such as 'minus' and 'subtract' as well as 'the difference between'.

Children are now encouraged to **count between the smaller and the larger numbers**, rather than backwards from the larger number.

They draw their own number line, label the two numbers from the calculation then jump up to the larger number in jumps of 1 and 10. e.g. $17 - 11 =$

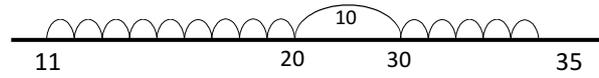


Starting at 11 and counting up to 17, children draw a jump on the line for every one they count. The jumps can then be counted to give the answer of 6. So $17 - 11 = 6$.

e.g. $35 - 11 =$

Children draw a line and label 11 and 35. They draw and count in steps of one to the next 10.

A large jump of ten can then be done before continuing in steps of one to the larger number.

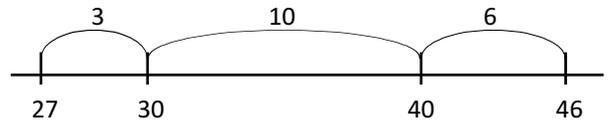


After reaching the larger number, the steps of ten and one can be counted to find the answer (the difference).

So $35 - 11 = 24$

As numbers become larger, children can count up in jumps of any size. Usually the first jump will be up to the next ten.

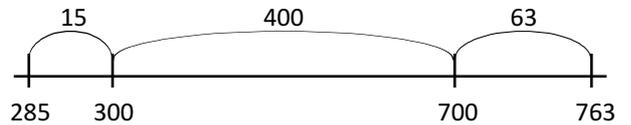
e.g. $46 - 27 =$



The jumps are then added:

$3 + 10 + 6 = 19$ so $46 - 27 = 19$

e.g. $763 - 285 =$



$15 + 400 + 63 = 478$ so $763 - 285 = 478$

Many children find they understand this approach and it is reliable and efficient for them.

An alternative method taught, which is a method that leads to the compact written method, is based on partitioning.

e.g. $48 - 23$

40	and	8
- 20	and	3
20	and	5

so $48 - 23 = 25$

And similarly, for $64 - 27 =$

60	and	4	50	and	14	5	1
- 20	and	7	60	and	14	5	1
			- 20	and	7	- 2	7
			30	and	7	3	7



Multiplication, X

Before standard multiplication is taught, children become familiar with counting objects in groups of two and groups of ten. When children are ready, they will count objects in groups of five.

Number sequences can be written e.g.

0, 2, 4, 6, 8, 10, 12, 14

0, 10, 20, 30, 40, 50, 60

0, 5, 10, 15, 20, 25, 30 etc.

Some numbers may be missed to encourage children to spot a pattern e.g.

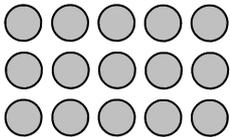
0, 5, __, 15, 20, 25, __, 35

The vocabulary 'lots of' will be used alongside the x sign to help progression to standard multiplication.

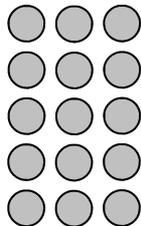
The words 'multiply' and 'times' are introduced as children are shown 'arrays'.

Arrays are a way to visually represent multiplication.

e.g. 3×5 (three rows of 5) 5×3 (five rows of three)



...will total 15.



Multiplication may also be presented as 'repeated addition'.

e.g. 3×5 is the same as $5 + 5 + 5 = 15$

These methods are used to help understand what multiplication involves.

Children are then expected to learn multiplication facts as well as associated division facts ($3 \times 5 = 15$, $15 \div 3 = 5$).

Children are usually introduced to times tables in the following order: 2, 10, 5, 3, 4, 6, 7, 8, 9, 11, 12

As learning progresses, multiplication and division facts become increasingly used in almost all areas of maths. Learning these facts by heart is therefore important. Please encourage your child to rehearse their times tables so they can recall the facts quickly.

The 'Grid Method'

You may hear your child say that they use the 'grid method'. This approach to solving multiplication problems is based upon the partitioning of numbers into 'tens and units' and 'hundreds tens and units'. It is used for larger calculations that cannot be done mentally.

Children still need to recall multiplication facts but the calculation is done in stages.

e.g. 16×5

$10 \times 5 = 50$

$6 \times 5 = 30$

$50 + 30 = 80$

$16 \times 5 = 80$

The 16 will be partitioned (split up) into 10 and 6 and then each part multiplied by 5. The 'products' are then added.

The grid follows the same principle as above but allows calculations to be organised more clearly.

The grid method; 16×5

x	10	6	
5	50	30	= 80

The grid method; 256×8

x	200	50	6	
8	1600	400	48	= 2048

The grid method; 24×36

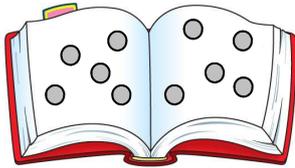
x	20	4	
30	600	120	720
6	120	24	<u>144</u> +
			<u>864</u>



Division, ÷

Initially division is used only as part of the every day environment in general activities. The main focus is upon sharing items between people and giving 'half'.

When division begins to be taught in maths lessons, it is done practically through sharing objects. Firstly, children are taught to share into two groups, and so find half.



Children may share into 2 groups using the pages of their book. A counter is put onto each page in turn until all the counters are shared. The children then count how many are on one page, which is half.

The answer will be written as 'half of 10 is 5' and later as '½ of 10 = 5'.

Sharing into four will likely be the next step, but follows the same principle shown above. A brief link will be made with fractions as a quarter (¼) and three quarters (¾) are found.

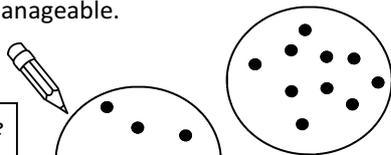
At this stage, the division symbol (÷) is not used.

The next stage of division will explicitly use the term 'divide' and the division symbol (÷) will be introduced.

A lot of division will be by 2, 5 and 10, which links with the initial multiplication tables used. 'Sharing circles' will be used whilst numbers are still manageable.

e.g. $20 \div 2 =$

Two sharing circles are drawn. Counting from 0 to 20, a dot is drawn in each circle in turn.



When the last dot is put in (number 20), the dots in one circle are counted.

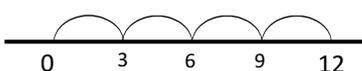
There are 10 dots in each circle so...

$$20 \div 2 = 10$$

Division by 5 will need 5 sharing circles, by 3 will need 3 circles and so on.

For division of larger numbers, a number line will be used.

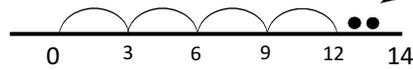
The children draw a blank number line and label the number to be divided at the far end e.g. $12 \div 3 =$



12 is labelled at the far end and from 0, jumps of 3 are made with the totals underneath.

Four jumps of 3 were made, so $12 \div 3 = 4$.

This method, unlike sharing circles, can be used for numbers that give rise to remainders, i.e. numbers that do not divide equally e.g. $14 \div 3 =$



Counting from 0 in 3s is done as before, but when another 3 is not possible, the remaining numbers are put as dots (i.e. 13 and 14)

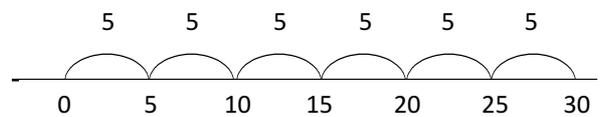
Four jumps of 3 were made with 2 left over

$$\text{so } 14 \div 3 = 4 \text{ r } 2$$

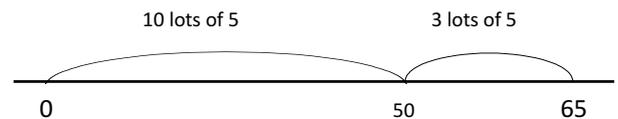
(four groups of 3 with a remainder of two)

The same method is continued until it can be used confidently. Children may find it helpful to write the number that is being counted on above the jumps.

e.g. $30 \div 5$



Eventually children will realise several 'lots of 5' (chunks) can be added (counted on) in one go; e.g. $65 \div 5$

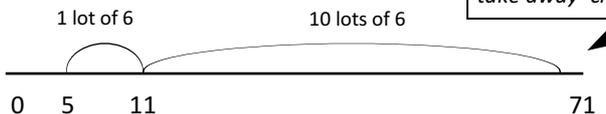


$65 \div 5$ can also be solved by 'taking away' 5, followed by another 5, followed by another 5, ...

We want children to use their knowledge of multiplication facts to realise that it is more efficient to subtract '10 lots of 5' and notice there is 15 remaining to subtract (which is 3 lots of 5). Since 'chunks' of the number are subtracted, this method is known as 'chunking'.

The same principle applies with numbers giving rise to remainders.

e.g. $71 \div 6$



Children begin at this end as they take away 'chunks'

$$\text{So... } 71 \div 6 = 11 \text{ remainder } 5$$

This subtraction idea then moves to:-

$$\text{e.g. } 65 \div 5 = 13$$

$$\text{e.g. } 71 \div 6 = 11 \text{ r } 5$$

65

71

- 50 (10 lots of 5)

- 60 (10 lots of 6)

15

11

- 15 (3 lots of 5)

- 6 (1 lot of 6)

0

5 remainder



Advanced Methods

This supplement shows the calculation methods that will be taught and used in school **after** the simpler methods are mastered. It is **crucial** that children have a **full understanding** of place value (the true value of digits in a number) before they begin to use these. Not all children will use these methods during their time in primary school.

Although the following are often quicker to use, they are not always the most appropriate for every maths problem. Children will therefore be encouraged to choose the most appropriate calculation method for themselves.

Addition, +

This method involves using columns in the same way as for the previous method and is known as 'the compact method' for addition. Children may refer to this as 'column addition'.

As previously, digits are aligned closely in columns.

Beginning with the units, the digits are added vertically. Where the total is greater than 10, the ten is carried into to next column.

$$\begin{array}{r}
 625 \\
 + 48 \\
 \hline
 3 \\
 1
 \end{array}$$

So here where $5+3=13$, the 3 remains in the units column and the 10 is 'carried' into the tens column.

$$\begin{array}{r}
 625 \\
 + 48 \\
 \hline
 673 \\
 1
 \end{array}$$

The tens are then added (2 tens + 4 tens and 1 ten) at the bottom, followed by the hundreds (in this case just 6 hundreds).

Where the number in a column totals ten or more, the number is carried into the next column to the left.

$$\begin{array}{r}
 645 \\
 + 578 \\
 \hline
 1223 \\
 11
 \end{array}$$

Here a hundred is carried into the hundreds column because $4\text{ tens} + 7\text{ tens} + 1\text{ ten} = 12\text{ tens}$. The number of hundreds is 12 so the thousands column is used.

Subtraction, -

'Column subtraction' requires a good understanding of the place value of numbers.

It is particularly useful when working with decimal numbers and money.

This method moves away from finding the difference on a number line and keeps the digits closely aligned in columns.

$$\begin{array}{r}
 95 \\
 - 58 \\
 \hline
 \end{array}$$

The largest number is written first. Subtraction is vertical and begins with the units.

$$\begin{array}{r}
 9^{15} \\
 - 58 \\
 \hline
 7
 \end{array}$$

Since 5 subtract 8 cannot be done, a **ten** is taken from the tens column to make the 5 into a 15 and leaving 8 tens in the tens column. $15 - 8 = 7$

$$\begin{array}{r}
 89^{15} \\
 - 58 \\
 \hline
 37
 \end{array}$$

The tens are then subtracted. 8 tens subtract 5 tens is 3 tens.

The same steps are followed for 3 digit numbers:

$$\begin{array}{r}
 2^{14}5 \\
 - 294 \\
 \hline
 051
 \end{array}$$

$5 - 4 = 1$
 $4\text{ tens} - 9\text{ tens}$ cannot be done so a **hundred** is taken from the hundreds column to make 14 tens. The hundreds column is adjusted. $14\text{ tens} - 9\text{ tens} = 5\text{ tens}$
 $2\text{ hundreds} - 2\text{ hundreds} = 0\text{ hundreds}$.

With money and decimal numbers the same principles are followed. The decimal points must also remain aligned.

$$\begin{array}{r}
 \pounds 2^{.12}5 \\
 - \pounds 1.93 \\
 \hline
 \pounds 1.32
 \end{array}$$

$5p - 3p = 2p$
 $20p - 90p$ cannot be done so a **pound** is taken to make 120p—
 $90p = 30p$.
 $\pounds 2 - \pounds 1 = \pounds 1$.



Multiplication, \times

This method for multiplication is probably the most difficult of the advanced methods and requires a thorough knowledge of multiplication and division facts (time tables). We also teach the expanded written method which uses partitioning.

The calculation is set out vertically as with column addition or subtraction. Here is an example of multiplication with 2 digit numbers. The 23 is multiplied first by the 8, then by the 30, before the answers are added.

$$\begin{array}{r} 23 \\ \times 38 \\ \hline 184 \\ \hline \end{array}$$

Firstly 3 is multiplied by 8 giving 24. The 4 units is put in the units column and the 4 tens is 'carried' into the tens column.

$$\begin{array}{r} 23 \\ \times 38 \\ \hline 184 \\ \hline \end{array}$$

Then 2 (the 20) is multiplied by 8 giving 16. The 2 carried into the tens column earlier is added on to make 18. The 8 goes into the tens column and the 1 into the hundreds.

$$\begin{array}{r} 23 \\ \times 38 \\ \hline 184 \\ \hline 0 \\ \hline \end{array}$$

Secondly the 23 is multiplied by the 30. For ease the 40 is thought of as 3 and a 0 is placed into the units column to begin (making the answer 10 times larger).

$$\begin{array}{r} 23 \\ \times 38 \\ \hline 184 \\ 690 \\ \hline 874 \end{array}$$

3 multiplied by 3 is 9. The 9 goes into the tens column.
3 multiplied by 2 is 6. The 6 goes into the hundreds column.

Finally the two answers are added using column addition = 881

Here is another example: $53 \times 36 =$

$$\begin{array}{r} 53 \\ \times 36 \\ \hline 318 \quad (6 \times 53) \\ 1590 \quad (\text{put } 0, 3 \times 53) \\ \hline 19108 \quad (\text{add answers: } 318 + 1590) \end{array}$$

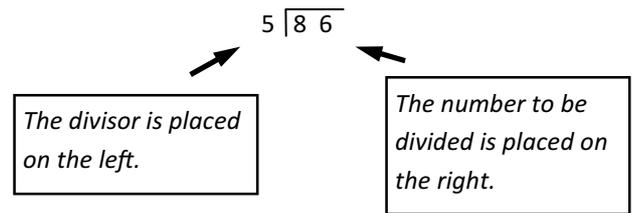
Division, \div

Division using 'chunking' as taught throughout the school is especially useful when dividing by 2 digit numbers.

However, when dividing by a single digit number, the following method of 'short division' can prove more efficient.

A thorough knowledge of multiplication and division facts (times tables) is **essential**.

This example shows how to use short division with 2 digit numbers, e.g. $86 \div 5 =$



Beginning with the tens this time, we ask "how many 5s in 8?". (It should be remembered that the '8' is actually 8 tens).

$$\begin{array}{r} 1 \\ 5 \overline{) 86} \end{array}$$

There is one 5 and 3 left over. The one goes above the 8 tens and the 3 (tens) then go across to the 6 to make 36.

$$\begin{array}{r} 17 \text{ r } 1 \\ 5 \overline{) 86} \end{array}$$

Then we ask "how many 5s in 36?". $7 \times 5 = 35$ so there are seven 5s and 1 left over.

So $86 \div 5 = 17 \text{ r } 1$

The 7 goes above the 35 and the remainder (r) is put next to the answer.

For larger numbers the same steps are followed.

e.g. $267 \div 5 =$

$$\begin{array}{r} 0 \\ 5 \overline{) 267} \end{array}$$

"How many 5s in 2 (hundreds)?" There are none so a 0 is placed above the 2 and the 2 (hundreds are placed next to the 6 (tens).

$$\begin{array}{r} 05 \\ 5 \overline{) 267} \end{array}$$

"How many 5s in 26(tens)?" There are five 5s and one left over so a 5 is put above the 26 and the 1 is placed next to the 7 to make 17.

$$\begin{array}{r} 053 \text{ r } 2 \\ 5 \overline{) 267} \end{array}$$

"How many 5s in 17?" There are three 5s and 2 left over so the 3 goes above the 17 and the 2 is a remainder.