Number sense in infancy predicts mathematical abilities in childhood

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Human infants in the first year of life possess an intuitive sense of number. This preverbal number sense may serve as a developmental building block for the uniquely human capacity for mathematics. In support of this idea, several studies have demonstrated that nonverbal number sense is correlated with mathematical abilities in children and adults. However, there has been no direct evidence that infant numerical abilities are related to mathematical abilities later in childhood. Here, we provide evidence that preverbal number sense in infancy predicts mathematical abilities in preschool-aged children. Numerical preference scores at 6 months of age correlated with both standardized math test scores and nonsymbolic number comparison scores at 3.5 years of age, suggesting that preverbal number sense facilitates the acquisition of numerical symbols and mathematical abilities. This relationship held even after controlling for general intelligence, indicating that preverbal number sense imparts a unique contribution to mathematical ability. These results validate the many prior studies purporting to show number sense in infancy and support the hypothesis that mathematics is built upon an intuitive sense of number that predates language.

analog magnitudes | approximate number system | cognitive development | mathematical cognition

Where does the uniquely human capacity for abstract mathematical concepts come from? What are the ontological building blocks that scaffold our ability for representing number symbolically and performing exact arithmetic? One hypothesis is that the preverbal, nonsymbolic numerical capacities exhibited by human infants in the first year of life serve as a conceptual basis for learning to count and acquiring symbolic mathematical knowledge (1, 2). Although many cognitive abilities contribute to math achievement, including working memory, inhibition, and other executive functions (3, 4), symbolic mathematics is commonly thought to build on a domain-specific nonverbal numerical representation (5). This system, the approximate number system (ANS), is an evolutionarily and ontogenetically ancient system that allows approximate representation of number without the need to count or rely on numerical symbols (2, 6).

Support for this hypothesis comes from a handful of studies that have shown a correlation between math ability and individual differences in ANS acuity. For example, ANS acuity in adolescence retroactively predicts math ability in elementary school (7) and ANS acuity in preschool-aged children correlates with their current and future math performance (8–12). In addition, children with dyscalculia, a severe deficit specific to math, have poorer ANS acuity than their typically developing peers (9, 11). These findings demonstrate that ANS acuity covaries with math ability, but the direction of this relationship remains unclear. One possibility is that ANS acuity guides the acquisition of the verbal counting system and symbolic math knowledge, such that children with greater ANS acuity learn to count earlier and have enhanced facility with spoken and written numerical symbols (13). Alternatively, learning the verbal counting system and early symbolic math concepts may refine ANS acuity, such that children with greater symbolic number knowledge end up with greater ANS acuity (13). To differentiate between these possibilities, it is necessary to investigate ANS acuity before children acquire the verbal counting system and before exposure to mathematics education. If the ANS is truly foundational for symbolic math, then early ANS acuity should inform children's facility with written and spoken numerical symbols.

Can the acuity of the preverbal number sense in the first year of life, well before an infant can count and understand verbal or written numerical symbols, predict later developing math abilities? A major stumbling block for answering this question has been the lack of parametric measures available for studying infant cognition in the first year of life. Our understanding of number sense in preverbal infants comes primarily from studies showing that groups of infants repeatedly shown pictures of the same number of objects (e.g., 8 dots) look longer when arrays with a new numerical value are presented (e.g., 16 dots) (14–17). These studies provide binary information (i.e., success vs. failure to discriminate between different numbers) about behavior at the group level but do not provide parametric scores of numerical sensitivity at the individual level. The development of a numerical change detection procedure that yields preference scores modulated by the ratio between numerical values in a changing visual stream provides a solution to this disconnect between infant and adult measures of numerical discrimination (18, 19). In the numerical change detection paradigm, infants observe two streams of visual arrays, one of which alternates between two numerical values while the other stays numerically constant and only changes in dot size and arrangement (Fig. 1*A* and Movie S1). Infants' preference for the numerically changing stream, as indexed

Significance

The uniquely human mathematical mind sets us apart from all other animals. How does this powerful capacity emerge over development? It is uncontroversial that education and environment shape mathematical ability, yet an untested assumption is that number sense in infants is a conceptual precursor that seeds human mathematical development. Our results provide the first support for this hypothesis. We found that preverbal number sense in 6-month-old infants predicted standardized math scores in the same children 3 years later. This discovery shows that number sense in infancy is a building block for later mathematical ability and invites educational interventions to improve number sense even before children learn to count.

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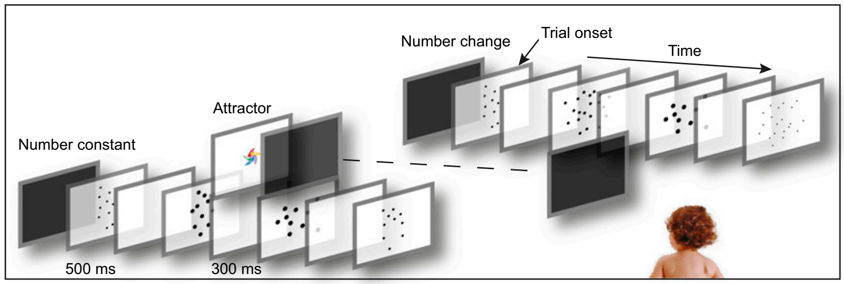
A



B

6 months			3.5 years		
Construct	**Test**	**Measure**	**Construct**	**Test**	**Measure**
ANS acuity	Numerical change detection task	Numerical preference score	ANS acuity	Non-symbolic number comparison task	Weber fraction (w)
Perceptual discrimination ability	Non-numerical change detection task (color or size)	Non-numerical preference score	Math ability	Test of Early Mathematics Ability (TEMA-3)	Standardized math score
			Counting knowledge	Give-A-Number task	Number knowledge (dichotomous)
			General intelligence (IQ)	Reynolds Intellectual Assessment Scales (RIAS)	IQ score

Fig. 1. (*A*) Schematic of the numerical change detection paradigm used to assess ANS acuity in infancy. The right side shows a numerically changing stream, which alternates between images containing 10 or 20 dots, while the left side shows a numerically constant stream, which in this example always contains 10 dots with variable dot sizes and arrangements. (*B*) Experimental design of the longitudinal study.

by longer looking times, is thought to reflect the acuity of their ANS, such that infants with better ANS acuity will show a greater preference for the numerically changing stream. At the group level, preference scores are parametrically related to the ratio between the numerical values in the changing numerical stream (18, 19). However, a particularly promising aspect of this dependent measure is that it is sensitive to individual differences between infants and can be used to track these differences longitudinally. Previous work has demonstrated that individual differences in ANS acuity remain stable during infancy: numerical preference scores at 6 months of age predict numerical preference scores at 9 months of age (18).

To address the relationship between infants' number sense and their later developing math abilities, we investigated the symbolic and nonsymbolic numerical abilities of 48 3.5-year-old children who had previously been tested in a numerical change detection study at 6 months of age. At 3.5 years of age, we assessed the children's numerical and mathematical understanding, as well as their general intelligence, using four widely used, age-appropriate measures (Fig. 1*B*). First, we tested children's ANS acuity using a nonsymbolic numerical comparison task in which they were asked to choose the numerically larger of two dots arrays. Based on performance on this task, we calculated a Weber fraction for each child (w, a measure of ANS acuity) using standard psychophysical modeling (7, 9, 12, 20–22). The value of w reflects the amount of noise in internal ANS representations, such that lower values of w correspond to more precise nonsymbolic numerical representations. Second, we assessed children's math ability using the Test of Early Mathematics Ability (TEMA-3) (23), which is a standardized math test designed for children as young as 3 years of age that consists of a series of verbally administered questions to assess counting ability, number-comparison facility, numeral literacy, and basic calculation skills. Third, children's knowledge of the verbal counting system was assessed using a variant of the Give-a-Number task (24) to determine the largest number word whose meaning the child understands exactly. Finally, children's general intelligence was assessed with the Reynolds Intellectual Assessment Scales (RIAS) (25), a standardized test that assesses both verbal and nonverbal intelligence with a series of verbally administered questions.

Results

Our main finding was that numerical preference scores from the numerical change detection task administered at 6 months of age significantly predicted both ANS acuity as measured by w ($r = -0.29$, $P = 0.02$) and standardized math scores ($r = 0.28$, $P = 0.03$) measured at 3.5 years of age (Fig. 2) (see *SI Results* for group scores on all tasks). (Several of the children who returned for the follow-up visits at 3.5 years of age had negative numerical preference scores at 6 months of age despite the fact that the mean score for each of the conditions from which they were drawn exhibited significant positive preference scores.) Note that because lower w scores are associated with greater ANS acuity, the negative correlation between numerical preference scores and w indicates that greater ANS acuity in infancy is associated with greater ANS acuity at 3.5 years. Critically, these relationships held even after controlling for general intelligence (w: $r_p = -0.30$, $P = 0.03$; math: $r_p = 0.35$, $P = 0.01$). In addition, we performed a multiple regression analysis to assess the contributions of numerical preference scores at 6 months of age and intelligence quotient (IQ) at 3.5 years of age to standardized math scores at 3.5 years of age. This model captured a significant amount of

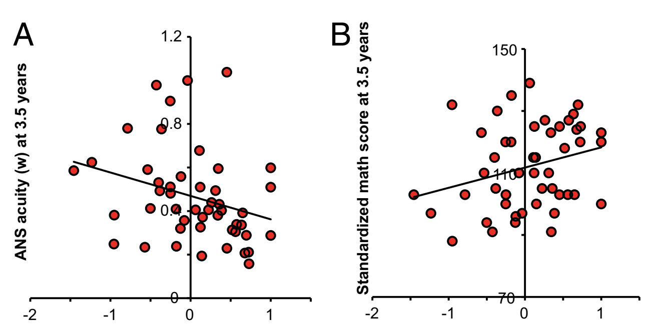


Fig. 2. Numerical preference scores in the numerical change detection task at 6 months of age are significantly correlated with ANS acuity as indexed by Weber fractions (w) (*A*) and with math ability (standardized math scores) (*B*) at 3.5 years of age.

variance in children's math achievement [R^2 = 0.28, $F_{(2,42)}$ = 8.18, P = 0.001] and both factors were unique predictors (numerical preference score: β = 0.31, P = 0.02; IQ: β = 0.45, P = 0.001). A second model assessing the contributions of numerical preference scores and IQ to ANS acuity at 3.5 years was marginally significant [R^2 = 0.13, $F_{(2,42)}$ = 3.06, P = 0.06] and numerical preference scores were a significant predictor (β = −0.30, P = 0.04), whereas IQ was not (β = −0.21, P = 0.16). These analyses suggest that the link between ANS acuity in infancy and math ability cannot be attributed solely to differences in general intelligence.

A median split based on math achievement scores also revealed that children with high math achievement scores (TEMA > 110) had significantly higher numerical preference scores than children with low math achievement scores (TEMA < 111) (P = 0.02). Children with high math achievement scores had numerical preference scores that were significantly greater than zero [$t_{(22)}$ = 2.06, P = 0.05], whereas children with low math achievement scores had preference scores that were not different from zero [$t_{(24)}$ = −0.87, P = 0.39].

To assess whether memory or general perceptual discrimination abilities in infancy could account for our results, we additionally examined the relationship between infant nonnumerical change detection scores and both w and standardized math scores at 3.5 years of age. One-half of the 6-month-old infants had also been tested on a nonnumerical version of the change detection task in which the color or size of a single shape varied in one visual stream and remained constant in the other. Preference scores in this task are thought to reflect short-term memory capacity (26) or perceptual discrimination ability and are independent of numerical preference scores (18). In this reduced sample, numerical preference scores, but not nonnumerical preference scores, were correlated with ANS acuity at 3.5 years (numerical preference scores: r = −0.42, P < 0.02; nonnumerical preference scores: r = 0.14, P = 0.25), and the correlation between numerical preference scores and ANS acuity was significantly greater than the correlation between nonnumerical preference scores and ANS acuity (z = −1.91, P < 0.03). However, there was no significant difference between the correlations for numerical versus nonnumerical preference scores and standardized math scores at 3.5 years of age (P > 0.2), and neither correlation reached significance (numerical preference scores: r = 0.16, P = 0.22; nonnumerical preference score: r = −0.02, P = 0.47). An important caveat is that in this reduced sample we may lack the statistical power to detect a true correlation between preference scores and math achievement. Given the significant correlation between numerical preference scores and childhood w, the lack of a correlation between numerical preference scores and childhood IQ, and the finding that numerical and nonnumerical preference scores are uncorrelated with each other both in the present sample and in previously published reports (18), it does not seem likely that the observed correlation between numerical preference scores and math achievement is driven by domain-general factors in infancy. Therefore, we believe that the most parsimonious conclusion is that ANS acuity in infancy, rather than domain-general memory or perceptual abilities, uniquely predicts later emerging math abilities and ANS acuity.

Numerical preference scores in infancy also predicted children's mastery of verbal counting principles. Children who understood the exact meaning of the number words "one" to "six" at 3.5 years of age had significantly higher numerical preference scores in infancy than children who only understood a subset of those count words [$t_{(38)}$ = −2.70, P = 0.01], even though the groups did not differ in general intelligence (P > 0.2).

Finally, consistent with previous findings, we found a significant concurrent link between w and standardized math scores (r = −0.42, P = 0.002), which extends prior findings of this correlation to 3.5-year-old children who have not yet begun formal math education. The relationship between children's math abilities and ANS acuity both measured at 3.5 years of age remained significant even after controlling for general intelligence (r_p = −0.35, P = 0.01).

Discussion

A fundamental question for researchers and educators alike is to understand the cognitive bases of uniquely human mathematical abilities. This longitudinal study was designed to probe the relationship between infants' nascent numerical representations and the acquisition of symbolic math knowledge in early childhood. Our results demonstrate that ANS acuity at 6 months of age is predictive of math achievement, number word knowledge, and ANS acuity at 3.5 years of age. Critically, these relationships hold after controlling for general intelligence in childhood. Although previous studies have provided evidence for a relationship between ANS acuity and math achievement (7, 9, 12, 21), they have been unable to address the origins of this relationship. By demonstrating that ANS acuity in infancy, long before the acquisition of number words or exposure to formal math instruction, predicts future math achievement, the present results suggest that symbolic arithmetic builds upon more primitive numerical representations.

Several possible explanations have been proposed to explain the mechanism underlying the relationship between ANS acuity and math performance. One hypothesis that has received some support from cross-cultural studies (27) is that exposure to and proficiency with symbolic number presentations sharpens ANS representations and improves acuity. An alternate hypothesis is that the ANS interfaces with arithmetic operations as a form of online error detection (28), such that sharper ANS acuity allows detection of erroneous symbolic answers. A third hypothesis is that greater ANS precision leads to improved facility with acquiring the meaning of number words and symbols and learning symbolic arithmetic (2, 5). Children with a sharper sense of quantity may be at an advantage for mapping numerical symbols to approximate magnitudes and for acquiring basic mathematical concepts. In support of this idea, recent studies have shown that experience with approximate arithmetic using nonsymbolic arrays leads to improvements in symbolic math performance (29, 30). In addition, the automaticity of ordering numerical symbols may mediate the relationship between ANS acuity and symbolic math performance in adults (31). Thus, heightened ANS acuity may aid children in the mapping of number words onto ANS representations and this may lead to earlier proficiency with numerical symbols and provide a jumpstart for math achievement.

Given that the correlation between ANS acuity and math achievement holds into adulthood (21, 22, 32), and given that adults in math-literate societies have on average slightly higher ANS acuity than adults from cultures without systematic count lists (27), it seems likely that there are bidirectional influences between ANS acuity and math achievement that continue throughout the life span. Nevertheless, the present finding that ANS acuity in infancy predicts number word knowledge and math scores in early childhood suggests that the influence of ANS acuity on math achievement precedes exposure to verbal counting and math education. This finding is therefore most consistent with the hypothesis that ANS acuity has a causal influence on math achievement.

However, despite the studies referenced above that find a relationship between ANS acuity and symbolic math, a number of other studies have found that ANS acuity either does not correlate with symbolic math achievement (33–35), that the relationship is mediated by executive function (36), or that the relationship holds only for children with low math ability (37). The presence of these discrepant findings indicates that the relationship between ANS acuity and symbolic math is not clear-cut, and further research is needed to elucidate how the ANS may serve as a scaffold for symbolic math skills. Given that the present findings concern children who are just beginning to acquire numerical symbols and symbolic arithmetic, one possibility is that it is at this point that the relationship between ANS acuity and burgeoning math knowledge is strongest (38).

It is also important to note that although we found that numerical sensitivity in infancy was a unique predictor of later math achievement, this relationship explained only a small proportion of the total variance. In fact, in many prior studies where individual differences in ANS acuity have been found to correlate with variance in math knowledge, ANS acuity was neither the only nor the strongest determiner of a child's math achievement (7, 8, 10, 36). Therefore, future research will need to investigate how ANS acuity relates to other factors that influence math achievement, and researchers will need to consider many potentially mediating variables when investigating the relationship between the ANS and symbolic math.

This longitudinal study provides evidence that preverbal number sense in infancy is predictive of both nonsymbolic number sense and symbolic mathematical ability in early childhood. Our uniquely human mathematical abilities appear to be fundamentally linked to an ontogenetically and evolutionarily ancient number sense that emerges in the first days of human life (39) and is ubiquitous throughout the animal kingdom (1, 40, 41). Although there may be bidirectional influences of ANS acuity and math ability over development, these data implicate a developmentally primary role for the preverbal number sense. This work may open the door for educational interventions to improve children's number sense even before they learn to count.

Materials and Methods

Subjects. Sixty-six infants who participated in a cross-sectional numerical change detection experiment (18) at 6 months of age (mean age, 6 months, 2 days) were rerecruited at 3.5 years of age. Seven children were excluded because they were unable to complete the nonsymbolic numerical comparison task at 3.5 years of age. Eleven children were excluded because w could not be modeled effectively (7, 9, 12). The final sample contained 48 children (mean age, 3.6 years; ranging from 3.5 to 3.9 years; 22 females).

Procedure. At 6 months of age, infants came into the laboratory for a single session during which the infant completed the numerical change detection task as part of one of five different cross-sectional studies. One-half of the infants also completed a nonnumerical change detection task, and the order of the two tasks was counterbalanced. Results from some of these cross-sectional studies have been previously reported (18), and additional details can be found in *SI Results*.

At 3.5 years of age, children came into the laboratory for two visits, each lasting approximately 1 h. Children were tested individually in a quiet room and were given small stickers throughout the session to maintain interest. During the first visit, children completed the TEMA-3 (23) and one session of the nonsymbolic number comparison task. During the second visit, children completed the verbal and nonverbal components of the RIAS (25), the counting knowledge task, and a second session of the nonsymbolic number comparison task. The order of the tasks within each session was counterbalanced across participants.

At each visit, parents gave written consent to a protocol approved by the local Institutional Review Board and were compensated monetarily and with a small gift for the child. After completing all three visits, families were given a $50 bonus.

Numerical Change Detection Task. Infants were shown two streams of images, one on each of two peripheral monitors. One of the image streams contained arrays that alternated in the number of elements (number changing stream) while the other image stream contained arrays with a constant number of elements (number constant stream). The cross-sectional studies varied in the number of elements presented in the changing and constant streams and the number of participants drawn from each of these cross-sectional studies varied as well: 6 vs. 24 ($n = 2$), 5 vs. 15 ($n = 18$), 6 vs. 18 ($n = 13$), 8 vs. 16 ($n = 2$), or 10 vs. 20 ($n = 13$). In all conditions, element size, cumulative contour length, cumulative surface area, and density were controlled across the two streams. For each infant, a numerical preference score was calculated by subtracting the proportion of time spent looking at the number constant stream from the proportion of time spent looking at the number changing stream. A positive preference score therefore indicates a preference for the changing stream, whereas a preference score of zero indicates no preference. To enable comparison of preference scores across numerical conditions, preference scores were normalized by dividing each score by the highest score in its respective condition.

Nonnumerical Change Detection Task. One-half of the infants performed either a color or size change detection task using the same procedure described above (size: $n = 17$; color: $n = 7$). In the color version, the constant stream displayed a single square with a constant color while the changing stream displayed a square that randomly changed between eight different colors. In the size version, the constant screen displayed a single Elmo face with a constant size, while the changing stream alternated between two Elmo faces that differed in size by a factor of 3. In these tasks, a nonnumerical preference score was calculated by subtracting the proportion of time spent looking at the constant stream from the proportion of time spent looking at the changing stream.

Nonsymbolic Numerical Comparison Task. On each trial, children were presented with two boxes (8 × 9.5 cm) on a touchscreen computer containing arrays of dots and were asked to touch the numerically larger array. Arrays contained between 4 and 14 elements, and the numerical ratio between the arrays was 1:2, 2:3, 3:4, or 6:7. To control for nonnumerical perceptual cues, the parameters of the arrays varied such that the smaller and larger numerical array each had the larger cumulative surface area on 50% of trials. All of the dots within a single array were homogenous in element size and color, and the color of each array varied randomly from trial to trial. Differential audiovisual feedback was provided after each trial, and children received a small sticker for each correct response to keep them engaged. Children performed practice trials until they made three consecutive correct responses or a maximum of 10 trials. Children performed 60 test trials in each test session for a total of 120 trials. Four children only have data from a single session due to a computer error ($n = 2$) or participating in only one session ($n = 2$).

To estimate each child's ANS acuity, we used a psychophysical modeling technique that has been used previously in the literature (7, 9, 12, 20–22) to calculate Weber fractions (w) based on the performance in the nonsymbolic numerical comparison task. We modeled the error rate as $\frac{1}{2}erfc\left(\frac{n_1 - n_2}{\sqrt{2}w\sqrt{n_1^2 + n_2^2}}\right)$, where n_1 is the numerosity of the larger set, n_2 is the numerosity of the smaller set, w is a measure of variance in the internal representation, and *erfc* is the complementary error function. The resulting value of w represents the noise in each participant's internal ANS representations, such that lower values of w correspond to less noise (i.e., higher ANS acuity). The model was unable to fit the performance of five children and settled on a very poor fit to the data ($r^2 < 0.2$) for six additional children. As in previous studies (7, 9, 12), these children were excluded from further analyses.

Counting Knowledge Task. This task was modeled after the Give-a-Number task (24). The experimenter introduced the child to a dinosaur puppet and asked the child to give the dinosaur a certain number of fish. On the first trial, the experimenter asked the child to give the dinosaur one fish. If the child successfully produced one fish, the child was asked to give the dinosaur three fish. If the child failed to produce one fish, the child was asked to give the dinosaur two fish. If the child provided the correct number of fish, the trials progressed in the order 1–3–5–6–6. If the child provided an incorrect number of fish for any number, the child was asked for $N - 1$ fish. The trials proceeded until the child answered correctly at least twice for N and failed at least twice for $N + 1$, or until the child successfully provided six fish twice. Children were grouped into two categories: those who understood the exact meaning of the number words "one" to "six" and those who did not. Data from 8 of the 48 children are missing because they were not administered the task.

Standardized Tests. Children's mathematical ability was assessed with the TEMA-3 (23), which consists of a series of verbally administered questions that assess age-appropriate counting ability, number-comparison facility, numeral literacy, and basic calculation skills. To assess general intelligence, children completed two verbal and two nonverbal subtests of the RIAS (25), and a composite IQ score was calculated for each child. RIAS scores from 3 of the 48 children are missing because they refused to complete all subtests.

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