

## **Appendix 1 Progression in Calculations Guide**

When teaching calculations in school it is expected that staff follow an agreed progression so that children do not become confused and that all children are moved on in their learning. This will be monitored by the Mathematics Leader each year.

The methods progress through year groups and children should only be taught the methods for their year group. Once children have mastered that method, they should be given opportunities to practice it intelligently by using and applying it in problem solving and investigative tasks.

Manipulatives must be used for **all** children to ensure everyone can access the learning and achieve the learning objective.

Staff must be familiar with all the methods and the progression for all year groups and Key Stages.

If at any time a class or group of children are finding a method difficult to grasp and manipulatives do not appear to be enabling children to access the learning, help and advice must be sought from the Mathematics leader and SEN leader as to how to progress.

Key number facts to note:

- Children should know their number bonds to 20 by the end of year 2.
- All multiplication facts (times tables) up to 12x12 should be known by heart by the end of year 4.
- Inverse operations must be referred to regularly to support fluency.

*EYFS stands for Early Years Foundation Stage.*

### Counting, Number and Place Value

<u>Year group</u>	<u>Objectives</u>	<u>Manipulatives and Strategies</u>
<b>EYFS</b>	<ul style="list-style-type: none"> <li>• count reliably with numbers from zero to twenty</li> <li>• place numbers in order</li> <li>• recognise the numerals</li> </ul>	Numicon Digit cards Counters Numerals
<b>1</b>	<ul style="list-style-type: none"> <li>• count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number</li> <li>• count, read and write numbers to 100 in numerals;</li> <li>• read and write numbers from 1 to 20 in numerals and words.</li> </ul>	Numicon Digit cards Counters Numerals Counting stick
<b>2</b>	<ul style="list-style-type: none"> <li>• recognise the place value of each digit in a two-digit number (tens, ones)</li> <li>• identify, represent and estimate numbers using different representations, including the number line</li> <li>• compare and order numbers from 0 up to 100; use <math>&lt;</math>, <math>&gt;</math> and <math>=</math> signs</li> <li>• read and write numbers to at least 100 in numerals and in words</li> <li>• use place value and number facts to solve problems.</li> </ul>	Numicon Digit cards Counters Numerals Counting stick
<b>3</b>	<ul style="list-style-type: none"> <li>• recognise the place value of each digit in a three-digit number (hundreds, tens, ones)</li> <li>• compare and order numbers up to 1000</li> <li>• identify, represent and estimate numbers using different representations</li> <li>• read and write numbers up to 1000 in numerals and in words</li> </ul>	Numicon Digit cards Counters Numerals Counting stick Dienes rods
<b>4</b>	<ul style="list-style-type: none"> <li>• count backwards through zero to include negative numbers</li> <li>• recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)</li> <li>• order and compare numbers beyond 1000</li> </ul>	Numicon Digit cards Counters Numerals

	<ul style="list-style-type: none"> <li>• identify, represent and estimate numbers using different representations</li> <li>• round any number to the nearest 10, 100 or 1000</li> <li>• solve number and practical problems that involve all of the above and with increasingly large positive numbers</li> </ul>	Counting stick Place Value counters
<b>5</b>	<ul style="list-style-type: none"> <li>• read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit</li> <li>• count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</li> <li>• interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers through zero</li> <li>• round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000</li> <li>• solve number problems and practical problems that involve all of the above</li> <li>• read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.</li> <li>• read Roman numerals to 1000 (M) and recognise years written in Roman numerals.</li> </ul>	Counters Numerals Counting stick Place Value counters
<b>6</b>	<ul style="list-style-type: none"> <li>• read, write, order and compare numbers up to 10 000 000 and determine the value of each digit</li> <li>• round any whole number to a required degree of accuracy</li> <li>• use negative numbers in context, and calculate intervals across zero</li> <li>• solve number and practical problems that involve all of the above.</li> </ul>	Counters Numerals Counting stick Place Value counters

### Addition and Subtraction

Year group	Method	Manipulatives and Strategies
EYFS Number	<p>To compare quantities of objects in groups, using the vocabulary 'more', 'fewer' and 'less'.</p> <p>Through the order of numbers to 20, be able to identify bigger/larger/smaller numbers than any given number.</p> <p>To know one more and one less than a given number.</p> <p>Using quantities and objects, to combine two groups of objects by counting them all and to remove a number of objects from the first group and work out how many are left by counting the remainder.</p> <p>To combine two or more groups of objects by counting on from the number in the first set.</p> <p>To add two single digit numbers by counting on to find the answer.</p> <p>To subtract two single digit numbers by counting back to find the answer.</p> <p>To record, using marks, which can be interpreted and explained.</p>	<p>Numicon</p> <p>Counters and tens frames</p> <p>Any other concrete resource for combining sets.</p>
1	<p>Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.</p> <p>Represent and use number bonds and related subtraction facts within 20.</p> <p>Add and subtract one-digit and two-digit numbers to 20 ,including zero</p> <p>Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as <math>12 = \square + 9</math>.</p>	<p>Numicon</p> <p>Counters and tens frames</p> <p>Dienes</p> <p>Pictorial representations</p>
2	<p>Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</p> <p>Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:</p> <ul style="list-style-type: none"> <li>a two-digit number and ones</li> <li>a two-digit number and tens</li> <li>two two-digit numbers</li> <li>adding three one-digit numbers</li> </ul>	<p>Numicon, Dienes and counters and pictorial representations, including those involving numbers,</p>

	<p>47+34</p> $\begin{array}{ccc} & \text{+4} & \text{+30} \\ \hline 47 & 51 & 81 \end{array}$ <p>36-17=</p> $\begin{array}{cccc} & \text{-10} & \text{-1} & \text{-6} \\ \hline 19 & 29 & 30 & 36 \end{array}$ <p>Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.  Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.  Recording should also be photographs of use of manipulatives and written number calculations (sentences).</p>	<p>quantities and measures.</p>
<p>3</p>	<p>Add and subtract numbers mentally, including:</p> <ul style="list-style-type: none"> <li>○ a three-digit number and ones</li> <li>○ a three-digit number and tens</li> <li>○ two three -digit numbers</li> </ul> <p>Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</p> <p>Partitioned vertical method:</p> $\begin{array}{r} 20 + 3 \\ 30 + 2 \\ \hline 50 + 5 = 55 \end{array}$ $\begin{array}{r} 50 \\ \cancel{60} \text{ } 13 - \\ \hline 20 \quad 7 \\ \hline 30 \quad 6 = 36 \end{array}$	<p>Numicon, Dienes and counters and pictorial representations, including those involving numbers, quantities and measures.</p>

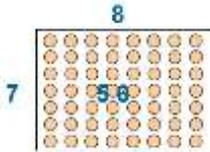
	<p>Units to be subtracted first, then tens and so on.  Estimate the answer to a calculation and use inverse operations to check answers  Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.</p>	
4	<p>Find 1000 more or less than a given number  Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.  Estimate and use inverse operations to check answers to a calculation.  Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.  Standard vertical method:</p> <p>introduce carrying</p> $\begin{array}{r} 625 \\ + 38 \\ \hline 663 \end{array}$ <p>introduce borrowing</p> $\begin{array}{r} 4516711 \\ 285 \\ \hline 286 \end{array}$	<p>Represent calculations using Cuisenaire rods, place value counters.  Numicon</p>
5	<p>Add and subtract whole numbers with more than 4 digits, including using formal written methods of (columnar addition and subtraction).  Add and subtract numbers mentally with increasingly large numbers  Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.  Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.</p>	<p>Cuisenaire Place value counters</p>

6	<p>Perform mental calculations, including with mixed operations and large numbers.</p> <p>Use their knowledge of the order of operations to carry out calculations involving the four operations</p> <p>Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.</p> <p>Solve problems involving addition and subtraction.</p> <p>Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.</p>	<p>Cuisenaire</p> <p>Place value</p> <p>counters</p>
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### Multiplication and Division

<b>Year Group</b>	<b>Method</b>	<b>Manipulatives and Strategies</b>
EYFS Number	<p>To solve problems through doubling single digits.</p> <p>Children are taught the concept of division through solving problems by sharing objects into equal groups and counting how many are in each group.</p> <p>To calculate half of a given number by sharing that quantity of objects into two groups and counting the numbers in each group.</p> <p>Even multiples up to 20.</p>	Numicon and counters
1	<p>Count in multiples of 2s, 5s and 10s.</p> <p>Up to 12x each number.</p> <p>Solve one-step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p>	<p>Numicon</p> <p>Double sided counters</p> <p>Coins</p> <p>Counting stick</p> <p>Arrays with double</p>

	<p>These should involve: sharing objects equally. Halving even numbers practically. Knowing halves of even numbers to 10 by heart.</p>	sided counters and objects.
2	<p>Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward. Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (<math>\times</math>), division (<math>\div</math>) and equals (<math>=</math>) signs. Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.</p> <p>**** (3x4=12) **** ****</p>  <p style="text-align: center;"><math>8 \div 2</math></p>	<p>Numicon Counting stick Arrays with double sided counters and objects.</p>
3	<p>Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</p> $\begin{array}{r l} 10 & 3 \\ 3 & 30 \end{array} 9 = 39 \text{ leading to } \begin{array}{r} 13 \\ \times 3 \\ \hline 39 \end{array} \text{ including carrying numbers.}$ <p>Mentally multiply by 9, 19, 29 etc by multiplying by the nearest multiple of 10 then adjusting. Solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are</p>	<p>Double sided counters Dienes Numicon Counting stick</p>

	<p>connected to m objects.</p> <p>Division: represent simple divisions as arrays, this then begins to show children how the short method is recorded. <b>However</b>, children MUST be taught to calculate division mentally using their multiplication facts when appropriate, before using the short written method.</p>  <p style="text-align: center;"> <math display="block">\begin{array}{r} 8 \\ 7 \overline{) 56} \end{array}</math> </p>	
4	<p>Recall multiplication and division facts for multiplication tables up to <math>12 \times 12</math></p> <ul style="list-style-type: none"> <li>count in multiples of 6, 7, 9, 25 and 1000</li> </ul> <p>Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.</p> <p>Recognise and use factor pairs and commutativity in mental calculations.</p> <p>Division: small numbers mentally using multiplication facts. For dividing 3 and 4 digit numbers by a single digit, the short written method is to be taught moving from no remainders to remainders.</p> <p>Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.</p> <p>Vertical method: <math display="block">\begin{array}{r} 13 \\ \times 3 \\ \hline 39 \end{array}</math> including carrying numbers.</p> <p>Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.</p>	<p>Double-sided counters</p> <p>Bar model</p> <p>Counting stick</p> <p>Cuisenaire</p>

<p>5</p>	<p>Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.          Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers          Establish whether a number up to 100 is prime and recall prime numbers up to 19</p> <p>Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers:</p> <p>Long multiplication: <math display="block">\begin{array}{r} 27 \\ \times 13 \\ \hline 81 \\ 270 \\ \hline 351 \end{array}</math> zero as a place holder</p> <p>Multiply and divide numbers mentally drawing upon known facts          Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context:</p> $\begin{array}{r} 046r2 \\ 7 \overline{)324} \end{array}$ <p>Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.          Recognise and use square numbers and cube numbers and their notation.</p>	<p>Double-sided counters</p> <p>Bar model</p> <p>Counting stick</p> <p>Cuisenaire</p>
<p>6</p>	<p>Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication (see Y5).</p>	<p>Double-sided counters</p> <p>Bar model</p>

Divide numbers up to 4 digits by a two digit whole number using the formal written method for long division , interpret remainders as whole number reminders, or by rounding , as appropriate for the context:

**Long division:**

$$\begin{array}{r} 031 \\ 16 \overline{)496} \end{array}$$

Divide numbers up to 4 digits by a two digit number using the formal written method of short division where appropriate, interpreting remainders according to the context (see Y5).

Perform mental calculations, including with mixed operations and large numbers

Use their knowledge of the order of operations to carry out calculations (BiDMAS).

Counting stick

Cuisenaire