Templewood School

Progression in Written Calculations

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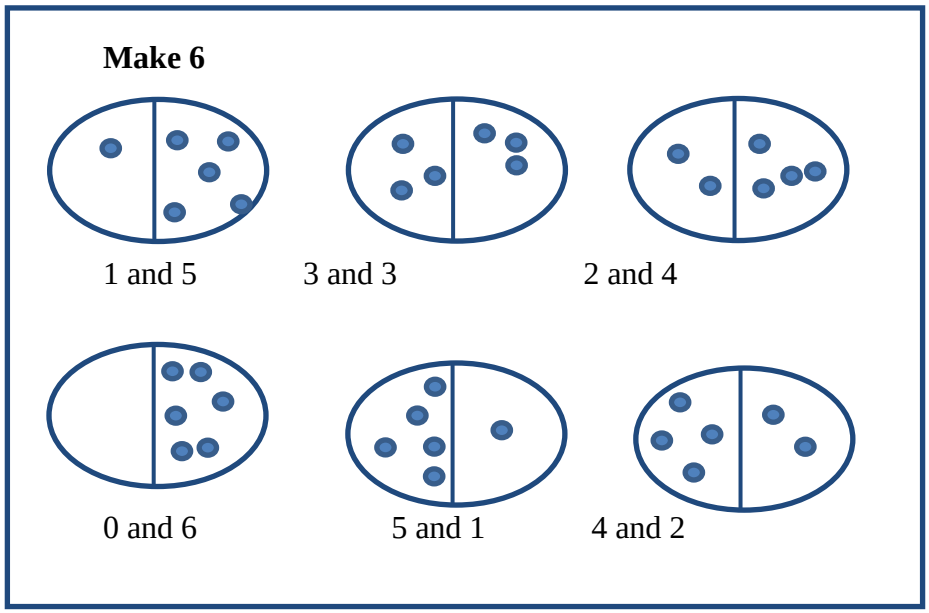
PROGRESSION THROUGH CALCULATION FOR ADDITION

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. ***The mental calculation strategies taught will continue to be used and developed and should not be replaced by written methods.***

The following stages are standards that we expect the majority of our children to achieve.

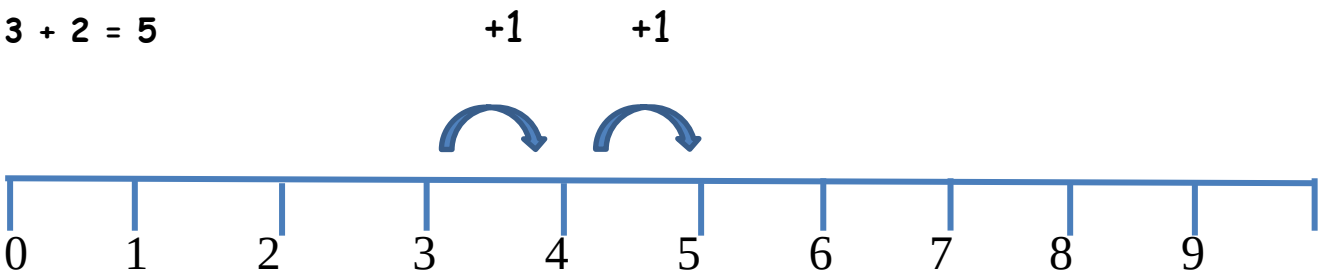
Step 1: Counting on U + U

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should develop ways of recording using pictures, etc. They use practical resources to support calculation and teachers demonstrate using a numbered line to count in ones.

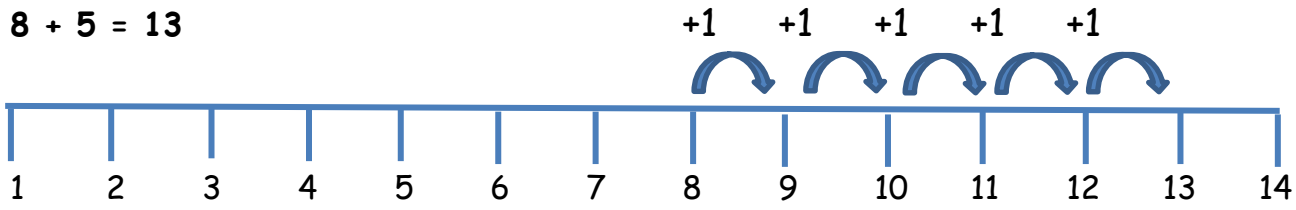


They use practical resources to support calculation and teachers demonstrate using a numbered line to count in ones.

3 + 2 = 5



Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

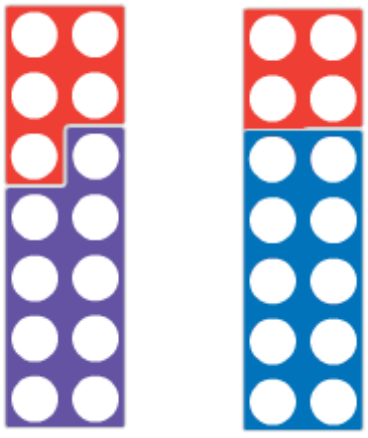
Addition with Numicon

Before children are ready to add they need to have had previous experience using Numicon and an awareness of the value of each Numicon piece.

4 + 2 = 6

Children find the Numicon pieces and combine to find the answer 6. They will then need to check they are correct by placing the 6 piece over the top.

When bridging ten children need to combine as normal, then place a ten piece over the top to clearly see how many tens and how many units. Children need to be aware of place value to 20 to aid them.

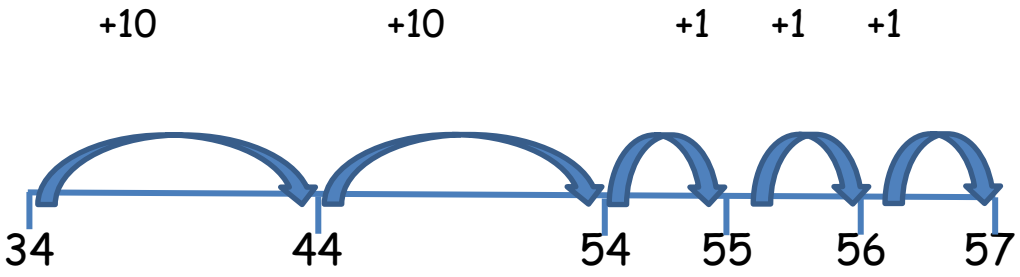


Step 2: Counting on TU + U

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

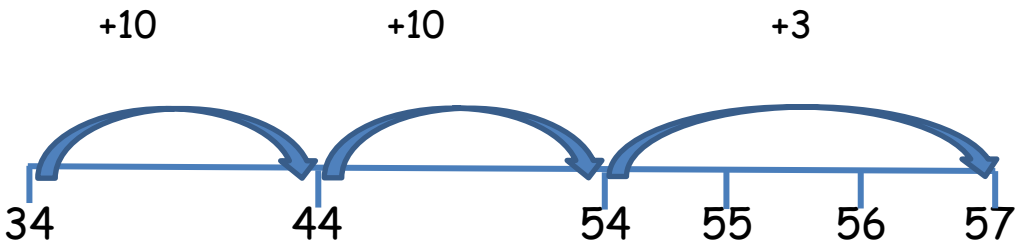
- First counting on in tens and ones.

34 + 23 = 57



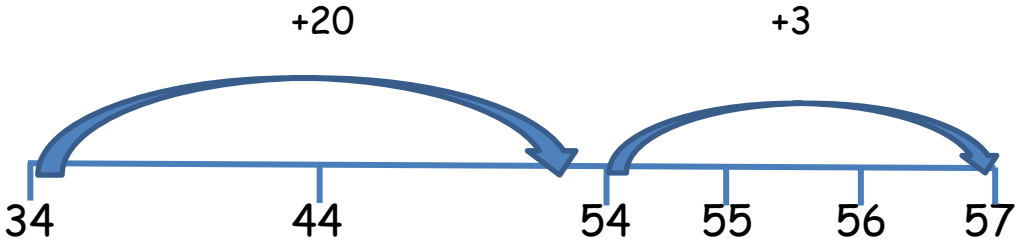
- Then helping the children to become more effective by adding the units in one jump (by using the know fact 4 + 3 = 7)

34 + 23 = 57



- Followed by adding the tens in one jump and the units in one jump.

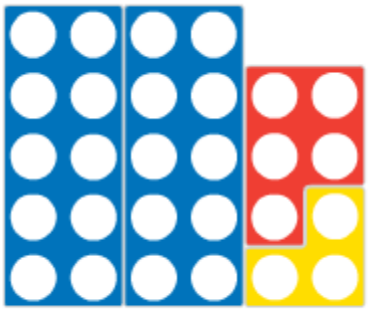
34 + 23 = 57

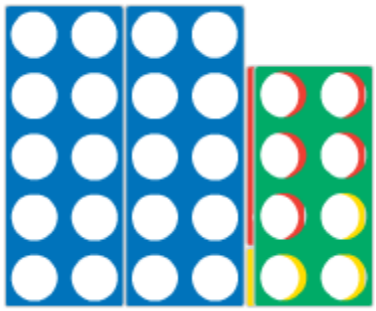


With numicon

If children do not recognise the units making 8 they may still need to place the 8 piece over the top to check.

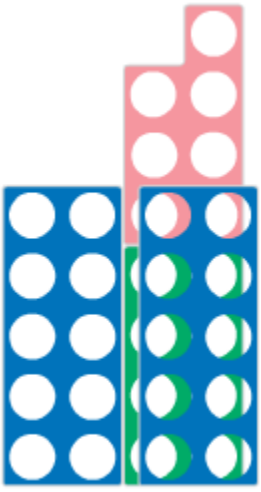
23 + 5 = 28





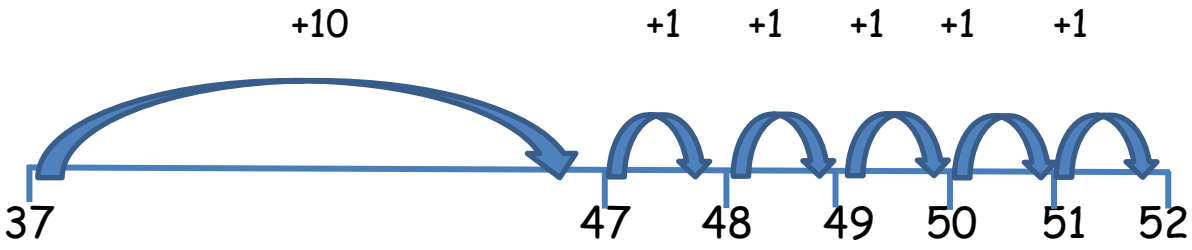
The idea of Numiconis for children to be able to recognise the shapes as a number. For the picture above children will see two tens and eight units which combined makes 28. Ensure the children count on from the starting number. For example, we made 23 at the beginning so keep that number in your head and count on 5.

17 + 8 = 25

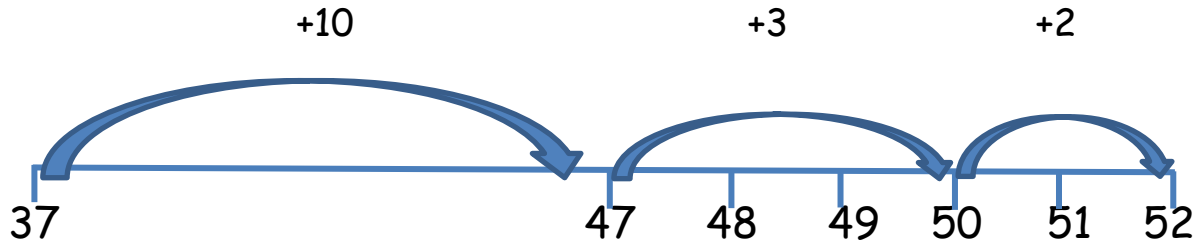


- Bridging through ten can help children to become more efficient.

37 + 15 = 52



37 + 15 = 52

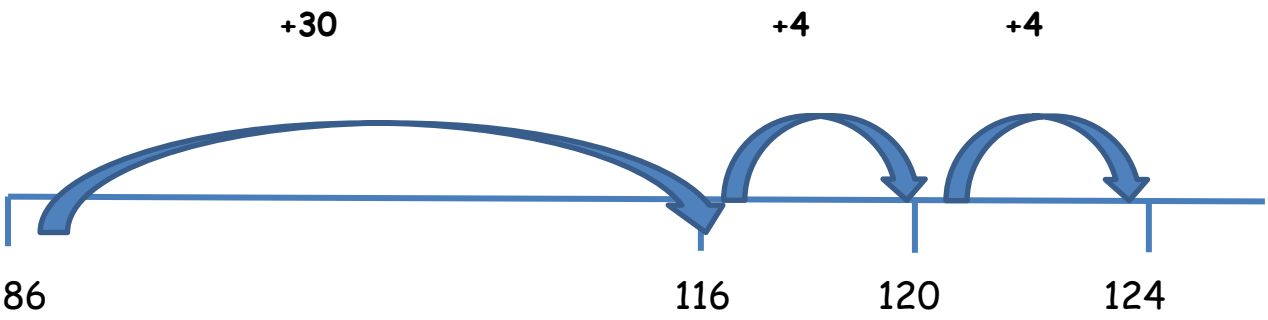


Step 3: Counting on TU + T

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

- Count on from the largest number irrespective of the order of the calculation.

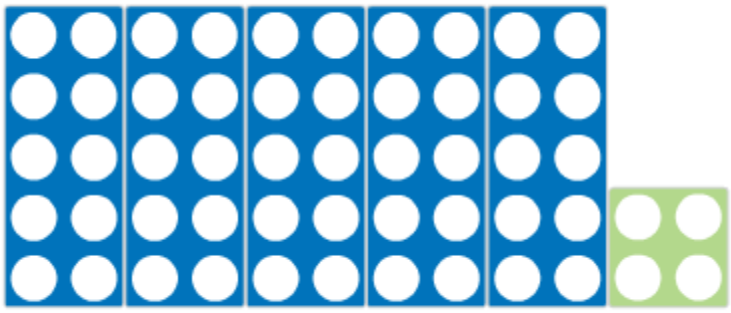
38 + 86 = 124



With nunicon

Make 34 first then add on the 20.

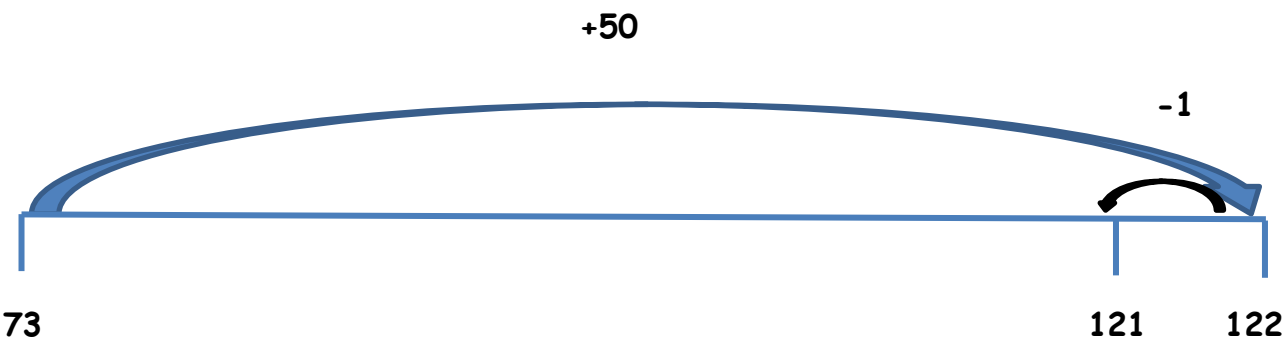
34 + 20 = 54



Step 4: Counting on TU + TU

- Compensation

49 + 73 = 122 **(73 + 50 -1)**



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on excising mental strategies.

Moving to adding the least significant digits first in preparation for 'carrying'.

Step 5: Partitioning

- (links to mental strategies)

47 + 76 =

7 + 6 = 13 40 + 70 = 110

110 + 13 = 123

47 + 76 =

↓

47 + 70 = 116

↓

116 + 6 = 123

Adding least significant digits first

	67	
+	24	
	11	(7 + 4)
	80	(60 +20)
	91	

	276	
+	85	
	12	(7 + 5)
	140	(60 + 80)
	200	
	352	

From this, children will begin to carry below the line (from Year 3)

$$\begin{array}{r} 625 \\ +\ 48 \\ \hline 678 \\ \hline 1 \end{array} \qquad \begin{array}{r} 367 \\ +\ 85 \\ \hline 452 \\ \hline 11 \end{array} \qquad \begin{array}{r} 3587 \\ +\ 675 \\ \hline 4262 \\ \hline 111 \end{array}$$

Using similar methods, children in Phase B will typically:

- ✓ *Add several numbers with different numbers of digits*
- ✓ *Begin to add two or more three-digit sums of numbers and money, with or without adjustment from the pence to the pounds*
- ✓ *Know that the decimal point should line up under each other, particularly when adding mixed amounts, e.g. £3.59 + 78p*
- ✓ *Begin to add two or more decimal fractions with up to three digits and the same number of decimal places*
- ✓ *Extend the carrying method to numbers with at least 4 digits.*
- ✓ *Children should extend the carrying method to numbers with any number of digits.*

(from Year 5)

$$\begin{array}{r} 3121 \\ +\ 37 \\ +\ 148 \\ \hline 3306 \\ \hline 1\ 1 \end{array} \qquad \begin{array}{r} 3.20 \\ +2.88 \\ \hline 6.08 \\ \hline 1 \end{array} \qquad \begin{array}{r} £3.48 \\ +\ £0.78 \\ \hline £4.26 \\ \hline 1\ 1 \end{array}$$

Using similar methods, children in phase C will typically:

- *add several numbers with different numbers of digits;*
- *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m + 280 cm.*

Using similar methods, children towards the end of Phase C will:

- *add several numbers with different numbers of digits;*
- *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.*

```
  401.20
+  26.85
+   0.71
  428.76
   1
```

```
  6584
+ 5848
 12432
 111
```

```
    42
  6432
   786
+ 4681
 11941
 1 2 1
```

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATION FOR SUBTRACTION

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. ***The mental calculation strategies taught will continue to be used and developed and should not be replaced by written methods.***

The following stages are standards that we expect the majority of our children to achieve.

Before children are ready to use Numicon and number tracks etc., they will need practical methods to solve simple subtractions. This may be using a range of objects or drawing pictures to support them.

6—3 = 3

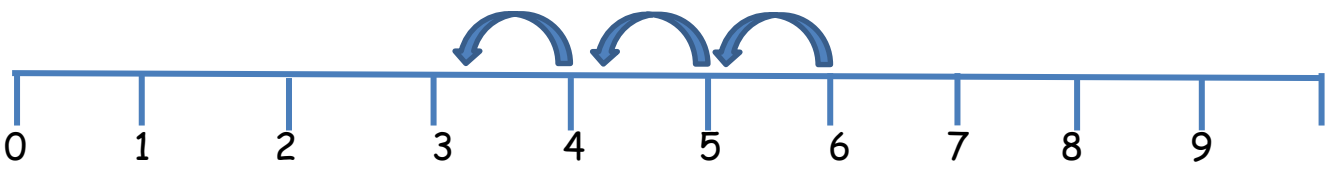
At this stage children will physically take away the objects to count how many are left.

Children are encouraged to develop a mental picture of the number system in their heads
to use for calculation. They develop ways of recording calculations using pictures etc.

Step 1: Counting back U -U
"Taking Away"

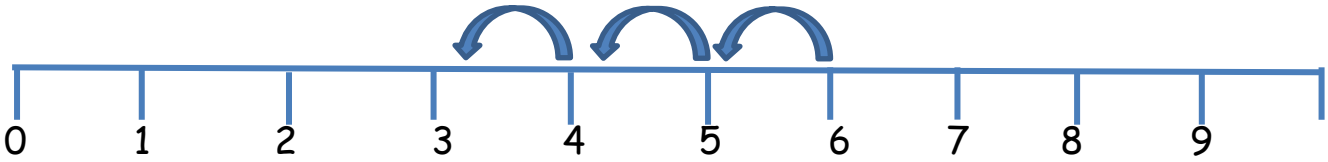
They use numbered lines and practical resources to support calculation. Teachers demonstrate the use of the number line.

6-3 = 3 **-1** **-1** **-1**



The number line should also be used to show that 6-3 means the 'difference between 6 and 3' or the 'difference between 3 and 6' and how many jumps they are apart.

6 – 3 = 3



Subtraction with numicon:

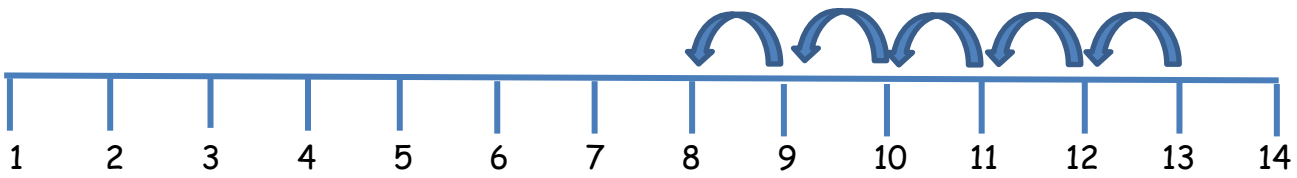
Children create the first number using Numicon. They take the Numicon piece for the amount they are subtracting and place this on top. Children should be able to recognise the shape left and check with a Numicon piece.

7—4 = 3

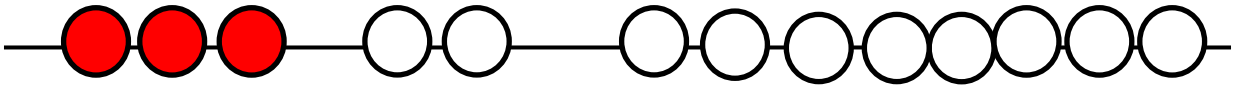
Step 2: Counting back TU -U
"Taking Away"

Children then begin to use numbered lines to support their own calculations – using a numbered line to count back in ones.

13 – 5 = 8 **-1 -1 -1 -1 -1**

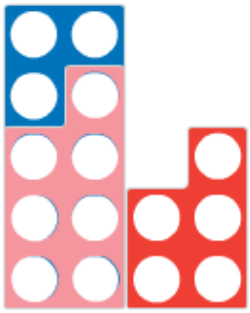


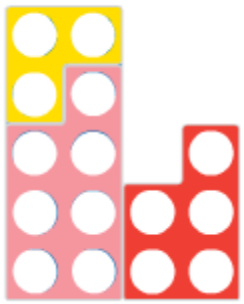
Bead strings or bead bars can be used to illustrate subtraction bridging through ten by counting back 3, then counting back 2.



Same as previously, making the two digit number first and cover with the piece you are taking away.

15 -7 =





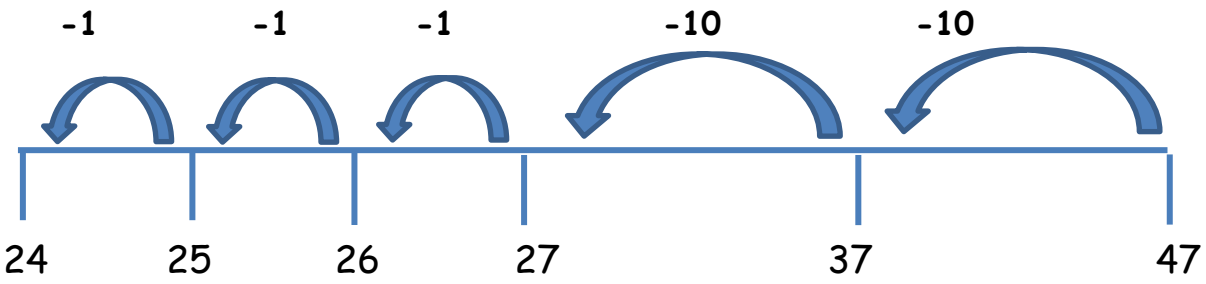
Step 3: Counting back "Taking Away"

·Counting back TU-U
·Counting back TU—T
·Counting back TU -TU

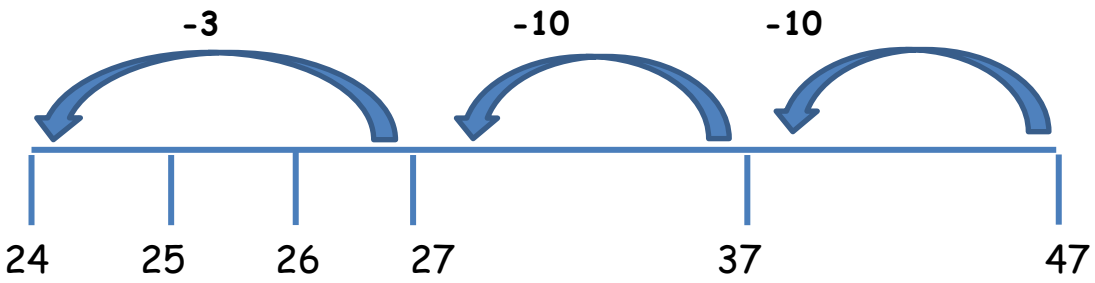
Children will begin to use empty number lines to support calculations.

- First counting back in tens and ones

47 – 23 = 24

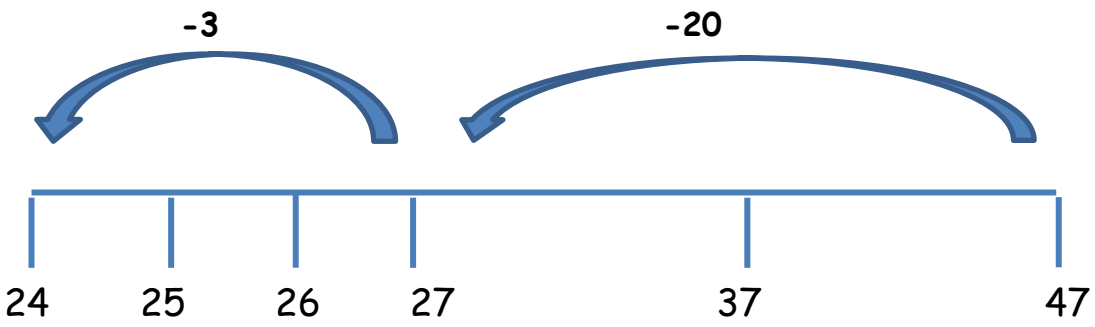


- Then helping the children become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4)



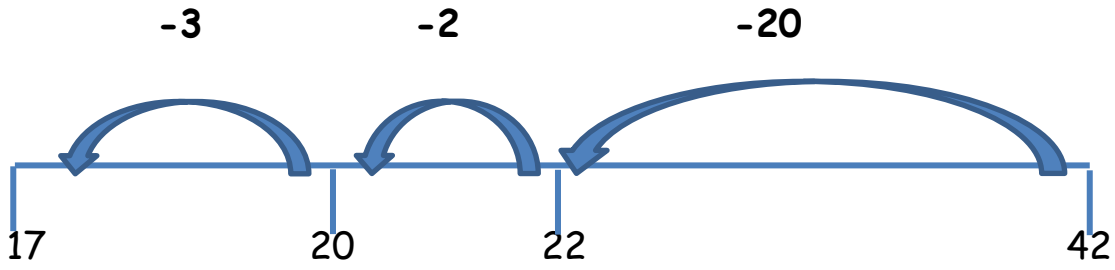
- Subtracting the tens in one jump and the units in one jump.

47 – 23 = 24



- Bridging through ten can help the children become more efficient.

42 – 25 = 17



"Taking Away" Using Hundred Square

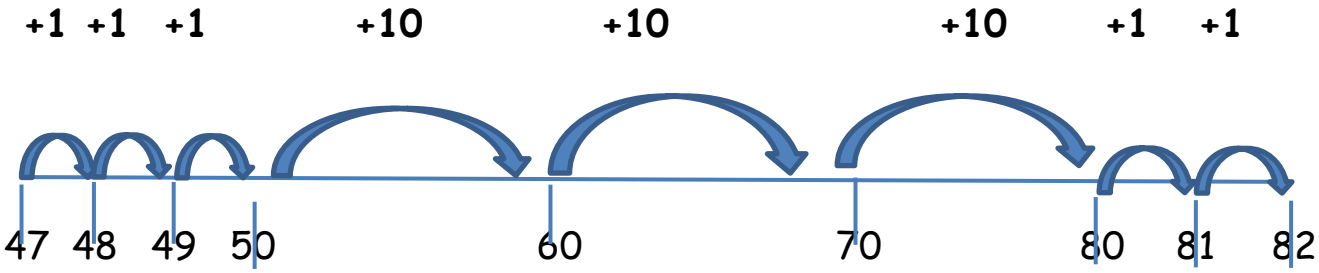
55 -23 = 32

This can be taught using a 100 square to count back. Find the starting number and jump up twice to represent two tens. Then jump back 3 spaces to represent the units.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 4: Subtracting by counting on "More than/Less Than"

82 – 47 = 35



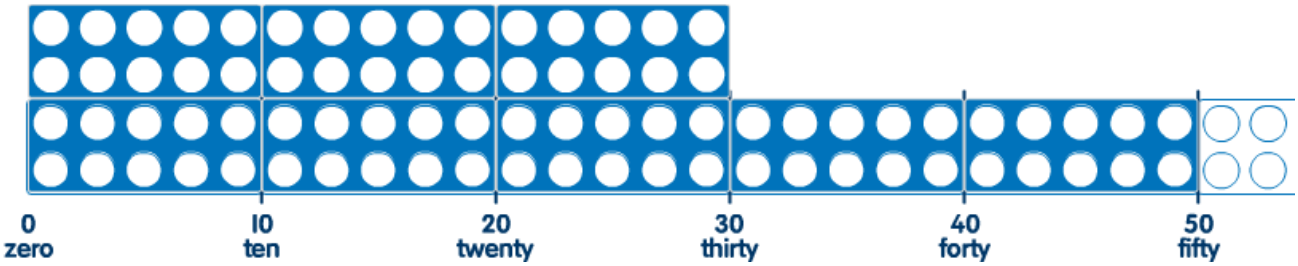
Children may count up in different ways which is fine as long as it is an efficient strategy.
Help children become more efficient with counting on by:

- Adding the units in one jump
- Adding the tens in one jump and the units in one jump
- Bridging through ten.

Children in Phase A will continue to use empty number lines with increasingly large numbers. Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Step 4: Subtracting by counting on using Numicon "More than/Less Than"

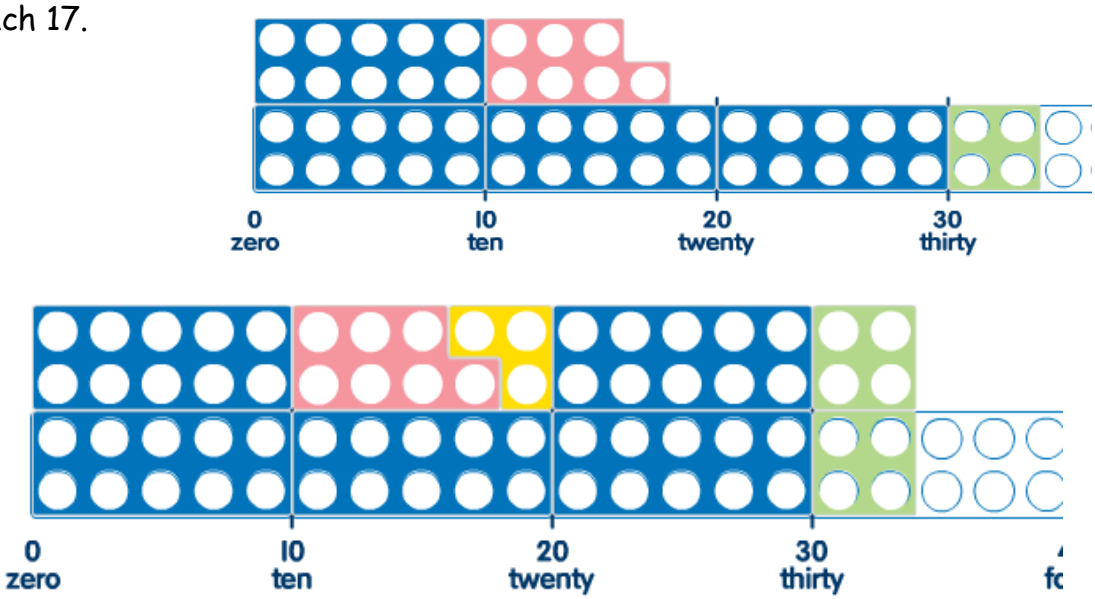
Set the scene for children.... Rosie has 50p and Isla has 30p. How much more does Rosie have? Children are to make the different amounts and place Isla's amount above Rosie's. Children will then be able to count on to get to Rosie's amount, finding the answer 20p.



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

"What is the difference"

Set the scene for children... Lizzie the lizard is 34cm long and Simon the slug is 17cm long. What is the difference in length between them? Children are to make each different amount and place Simon's length above Lizzie's. Children add to the next multiple of 10 (+3) and then to the end number (+14). Children finally combine 14 + 3 to reach 17.



Step 5: Partitioning

This process could be demonstrated using place value arrow cards to show partitioning and base 10 materials to show decomposition of the number.

- Initially the children will be taught examples that do not need exchange.

```
  8 9
- 5 7
-----
-----
```

```
  80  →  9
- 50  →  7
---------
  30  →  2 = 32
---------
```

- From this children begin to exchange.

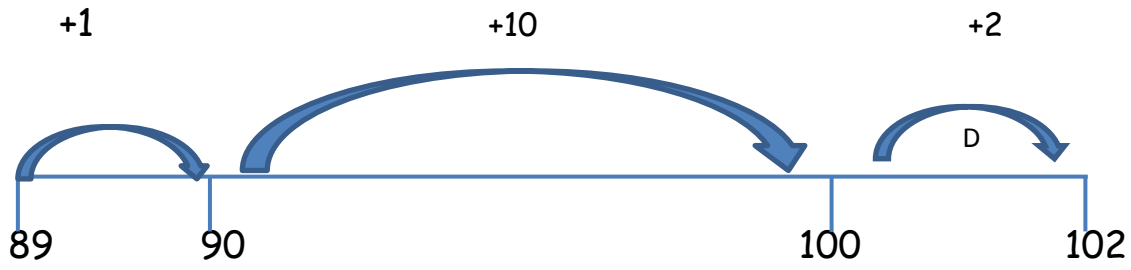
```
  71
- 46
----
----
```

```
   60      11
   7 0  →  1
 - 4 0  →  6
-------------
   2 0  →  5 = 2 5
-------------
```

Children should know that the units line up under units, tens under tens and so on.

- Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc. counting on using a number line should be used.

102 – 89 = 13



- Adjusting tens and units

$$\begin{array}{r} 794 \\ -\ 86 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{lccc} & \overset{80}{\cancel{90}} & \overset{14}{\cancel{4}} \\ 700 \rightarrow & \rightarrow & \rightarrow \\ & 80 & 6 \\ \hline 700 \rightarrow & 00 & \rightarrow 8 = 708 \\ \hline \end{array}$$

- Adjusting hundreds and tens

$$\begin{array}{r} 728 \\ -\ 86 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{lccc} & \overset{600}{\cancel{700}} & \overset{120}{\cancel{20}} & 8 \\ - & & 80 & 6 \\ \hline & 600 & 40 & 2 = 642 \\ \hline \end{array}$$

- Adjusting hundreds, tens and units

$$\begin{array}{r} 754 \\ -\ 86 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{lccc} & 600 & \overset{140}{\cancel{40}} & \overset{14}{4} \\ & \cancel{700} \rightarrow & \cancel{50} \rightarrow & 4 \\ - & & 80 \rightarrow & 6 \\ \hline & 600 \rightarrow & 60 \rightarrow & 8 = 668 \\ \hline \end{array}$$

Children in Phase B should:
- Be able to subtract numbers with different numbers up to 4 digit – 4 digits.
- Using this method children should also begin to find the difference between two three digit sums of money, with or without 'adjustment' from the pence to the pounds
- Know that decimal points should line up under each other.

For example:

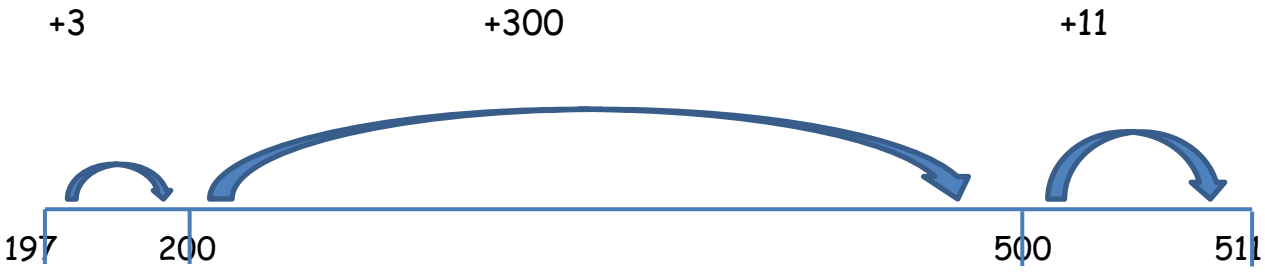
$$\begin{array}{r} £8.95 \\ -\ £4.38 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{lccc} & £8 & \overset{0.8}{\cancel{0.9}} & \overset{0.15}{\cancel{0.05}} \\ - & £4 & 0.3 & 0.08 \\ \hline & £4 & 0.5 & 0.07 = £4.57 \\ \hline \end{array}$$

Alternatively, children can set the amounts to whole numbers, i.e. 895 – 438 and convert to pounds after the calculation.

- Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

551 – 197 = 314



Stage 6: Decomposition

Initially the children will be taught examples that do not need exchange.

$$\begin{array}{r} 89 \\ -\ 57 \\ \hline 32 \\ \hline \end{array}$$

- From this children begin to exchange – adjusting tens and units.

$$\begin{array}{r} {\scriptstyle 6\ 1} \\ \not{7}\ 1 \\ -\ 46 \\ \hline 25 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 8\ 1} \\ 7\not{9}\ 4 \\ -\ 86 \\ \hline 708 \\ \hline \end{array}$$

- Adjusting hundreds and tens

$$\begin{array}{r} {\scriptstyle 6\ 1} \\ \not{7}\ 28 \\ -\ 86 \\ \hline 642 \\ \hline \end{array}$$

- Adjusting hundreds, tens and units

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 6\ 4\ 1} \\ \not{7}\not{5}\ 4 \\ -\ 86 \\ \hline 668 \\ \hline \end{array}$$

Children in Phase C should:
- Be able to subtract numbers up to 4 digits – 4 digits
- Using this method children should also begin to find the difference between two three digit sums of money, with or without 'adjustment' from the pence to the pounds, as well as subtracting any decimal numbers.
- Know that decimal points should line up under each other.

For example:

$$\begin{array}{rr} & 8\ 1 \\ & £8.\not{9}5 \\ - & £4.38 \\ \hline & £4.57 \end{array}$$

By the end of Year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. If calculating time, children should use a number line.

Children should not go onto the next stage if:

1. they are not ready.

2. they are not confident.

Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to check their answers after calculation using an appropriate strategy.

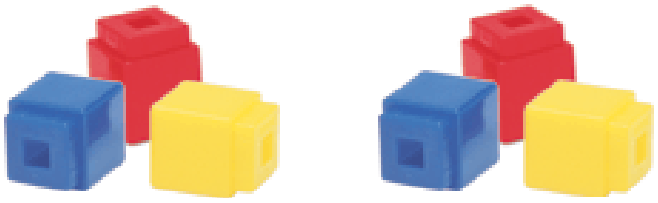
PROGRESSION THROUGH CALCULLTION FOR MULTIPLICATION

The aim is that children use mental methods when appropriate, but for Calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. ***The mental calculation strategies taught will continue to be used and developed and should not be replaced by written methods.***
The following stages are standards that we expect the majority of our children to achieve.

Step 1

Multiplication skills begin in the Early Years with counting in different steps.
Early stages of multiplication involve counting sets of objects to find the total. For example:

2 x 3 = 6
(2 sets of 3)



or

3 x 2 = 6
(3 sets of 2)

or counting in groups, e.g. counting in groups of two

2 **4** **6**

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.

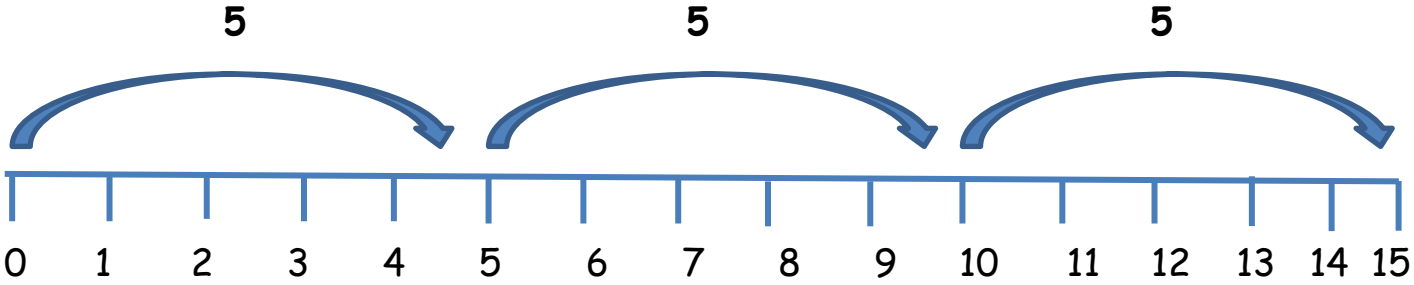
Step 2: Repeated addition to work out multiplication

Children will develop their understanding of multiplication and use jottings to support calculation:

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 3 x 5

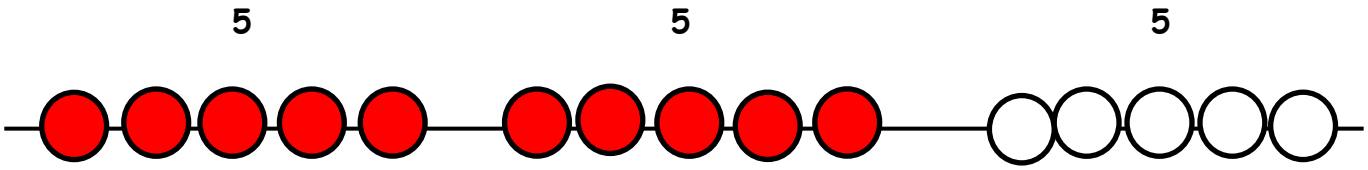
Repeated addition can be shown on a number line:

5 x 3 = 5 + 5 + 5



Repeated addition can be shown on a bead string or bead bar:

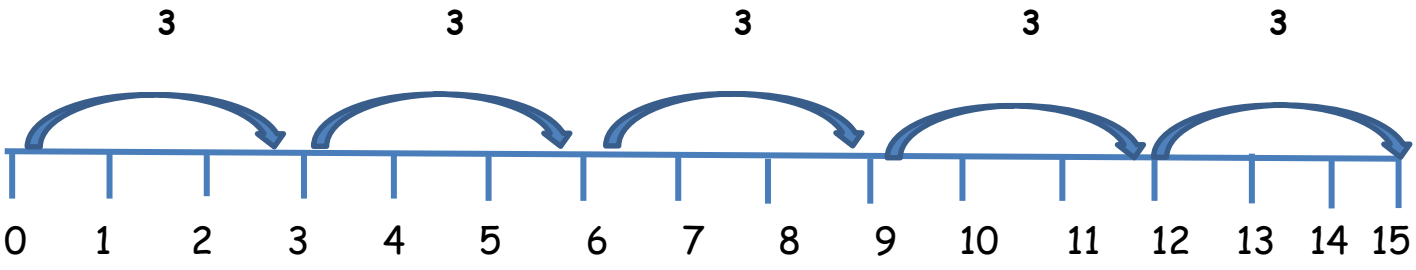
5 x 3 = 5 + 5 + 5



- **Commutatively**

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on a number line.

3 X 5 = 3 + 3 + 3 + 3 + 3



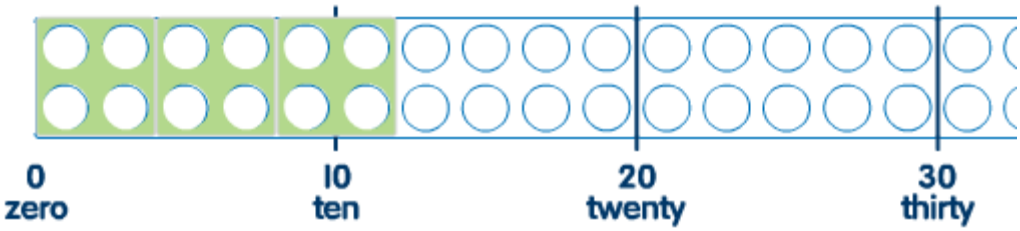
Repeated addition to work out multiplication using Numicon

3 x 4 =

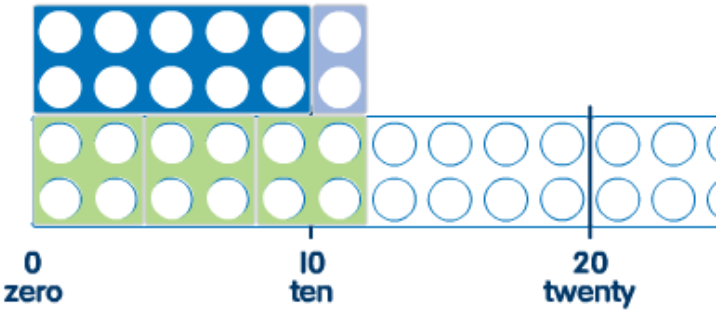
Children should now be aware that the calculation means we need 3 sets of 4. Children need to be taught that they are counting in 4's so will use the Numicon 4 piece. For this calculation they need three 4 pieces.

Children may be able to count on in 4's to work the answer out, if not steps can be put in place to help them to recognise the answer.

For example: using a Numicon number line.



Or placing Numicon (tens and units) pieces above the sets of 4.

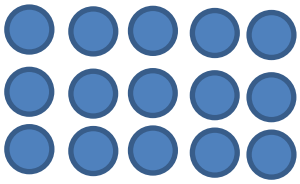


- **Arrays**

Children will need to practically make arrays using objects or see them as pictures to understand the process before drawing their own.

8 x ___ = ___

Ensure the same language as before is used "Here is one set, how many are in the set? How many sets are there?"



3 x 5 = 15

5 x 3 = 15

Record as arrays and be able to give related facts.

3 x 4 =

3 groups of 4 = 12

E.g. x x x
x x x
x x x
x x x

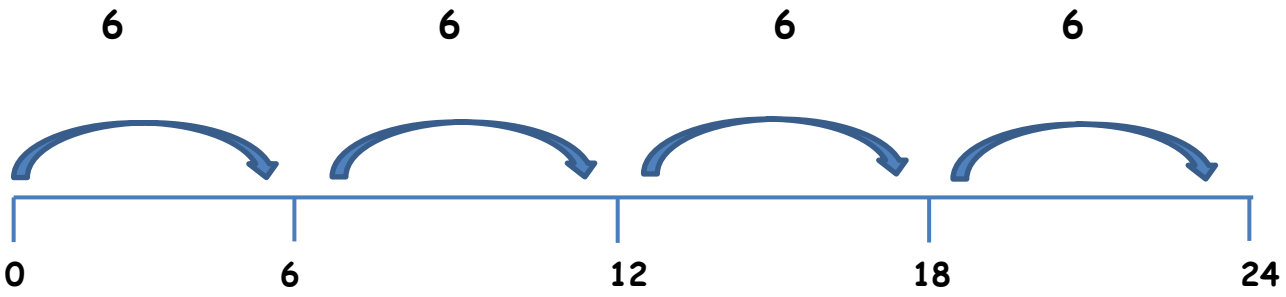
4 groups of 3 = 12

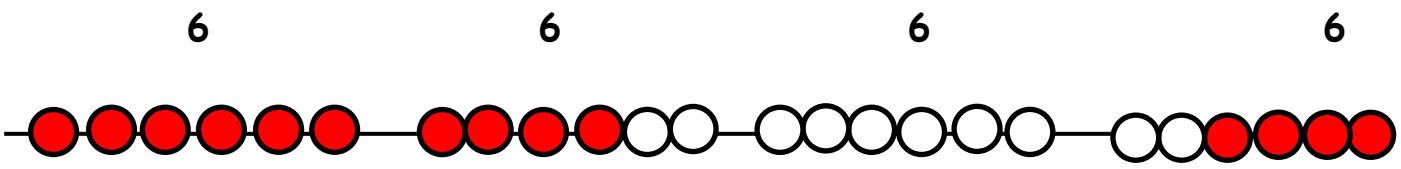
E.g. x x x x
x x x x
x x x x

Step 3: Children in phase A will continue to use repeated addition.

4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

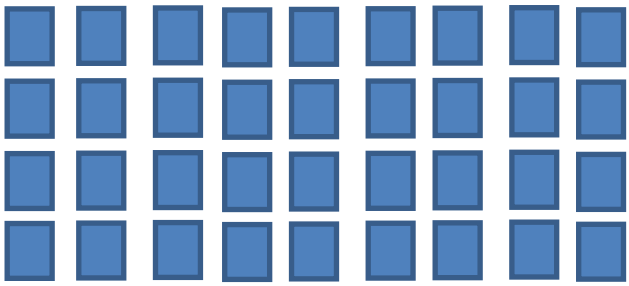
Children should use number lines or bead strings or bead bars to support their understanding.





- Arrays

Children should be able to model multiplication using an array.



4 x 9 = 36

9 x 4 = 36

- **Scaling**

Children should develop an understanding of scaling.
e.g. Find a ribbon that is 4 times as long as the blue ribbon.

5cm 20cm

- Using symbols to stand for unknown numbers to complete equations using inverse operations.

□ x 5 = 20 3 x △ = □ x ○ = 32

Step 4: Partitioning

38 x 5 = (30 x 5) + (8 x 5)
= 150 + 40
= 190

Step 5: Multiply a single digit number by a multiple of ten

- **Grid method: TU x U**

(Short multiplication – multiplication by a single digit)

2 x 40 =

	40
2	80

Step 6: Use grid method to multiply a 2 digit number by a single digit number

23 X 8 =

X	20	3
8	160	24

160
+ 24
184

Grid method using partitioning to multiply 2 digit numbers

27 X 43 =

X	20	8
40	800+	280
3	60+	21

=1080
=81 +
1161

Grid method using partitioning to multiply 3 digits by 1 digit

346 x 6

X	300	40	6
1800	240	36	

6
1800
240
+ 36
2076
1

Grid method using partitioning to multiply 4 digits by 1 digit

4346 x 8

X	4000	300	40	6
	16000	2400	320	48

8

32000
2400
320
+ 48
34768

Grid method using partitioning to multiply 3 digits by 2 digit

372 x 24

X	300	70	2
20	6000	1400	40
4	1200	280	8

7200
1680
+ 48
8928
1

Using similar methods, children will be able to multiply decimals with up to two decimal places by a single digit number and then a two digit number. They should know that the decimal points line up underneath each other.

For example: x **4** **0.9**

12	**2.7**

4.9 x 3 = **3**

$$\begin{array}{r} 12.0 \\ +\ \ 2.7 \\ \hline 14.7 \\ \hline \end{array}$$

4.92 x 3 = **x** **4** **0.9** **0.02**

12	**2.7**	**0.06**

3

$$\begin{array}{r} 12.00 \\ +\ \ 2.70 \\ \ \ 0.06 \\ \hline 14.76 \\ \hline \end{array}$$

Step 7: Vertical method for multiplication

There are 2 formal written column methods for multiplication in Phase C. In order for children to be able to make good progress with these methods they must have good recall of their times tables (up to 12 x 12).

27 X 43 =

$$\begin{array}{r} 27 \\ \times\ \ 43 \\ \hline 21\ (7 \times 3) \\ 60\ (3 \times 20) \\ 280\ (40 \times 7) \\ 800\ (20 \times 40) \\ \hline 1161 \end{array}$$

Children should extend this method to at least HTU x TU and to multiply decimals

$$\begin{array}{r} 72.0 \end{array}$$

```
  3.8
x
  1.6
 56.0
  6.0
210.0
273.6
```

Alternatively, children can set the amounts to whole numbers, i.e. 72 x 38 and convert to decimals after the calculation.

Step 9: Short vertical multiplication

23 x 8 =

```
  23
x  8
 184
 ~~2~~
```

Children should extend this method to at least THTU x U, HTU x TU and HTU x HTU and to multiply decimals

2.3 x 8=

```
  2.3
x  8
 18.4
 ~~2~~
```

Step 9: Long Multiplication

72 x 38 =

```
  72
X 38
 576
2160
2736
 ~~1~~
```

1. Take the ones first and multiply it by each of the columns above starting with ones (8 x 2), carry the tens and add to the next sum (8 x 70)

2. Now multiply each number above by the tens column (30 x 2) and (30 x 70)
You must place a 0 in the units column to make the answers 10 x bigger

3. Finally, starting with the units, add the columns together (576 + 2160)

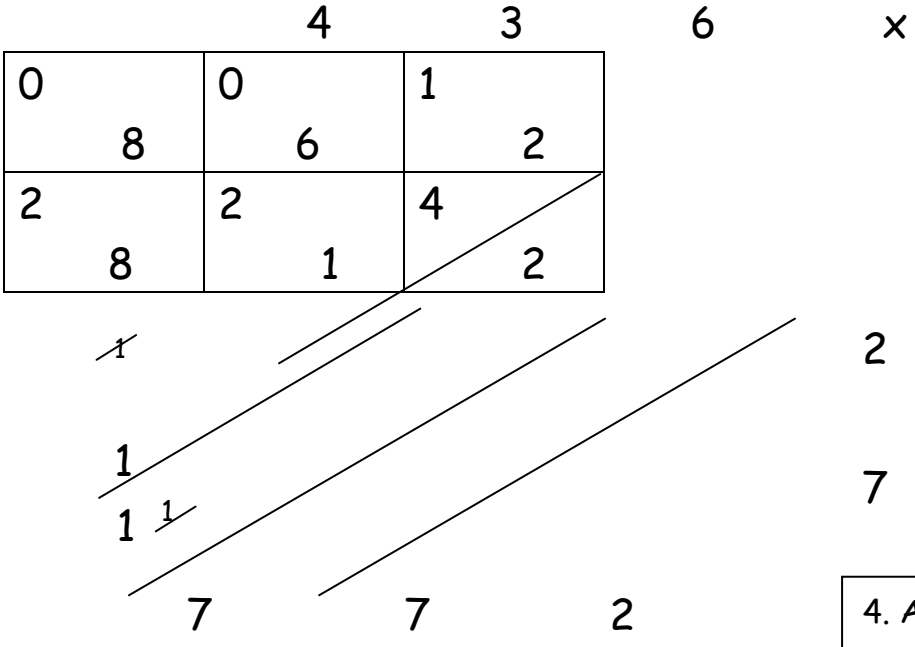
286 x 29

```
 286
X 29
2574
5720
8294
```

Using similar methods, children will be able to multiply decimals with up to two decimal places. They should know that the decimal points line up under each other.

Napiers bones:

John Napier was a Scottish mathematician and inventor. Napier is famous for creating mathematical logarithms, creating the decimal point, and for inventing Napiers bones – a calculating instrument. This alternative method is shared with Year 6 pupils because it's easy to use the format helps learners make good progress with their multiplication written calculations. This method is recognised at a higher level and many children go on to use this in secondary school.



1. Draw grid as shown above
2. You can start anywhere but one corner is best
3. If the answer is ones, then include 0 before e.g. 4 x 2 = 08

4. Add up each 'slide', working from right to left.
5. Where the numbers make 2 digit numbers carry the ten to the left. E.g. 1 + 6 + 2 + 8 =1

(ten is carried)

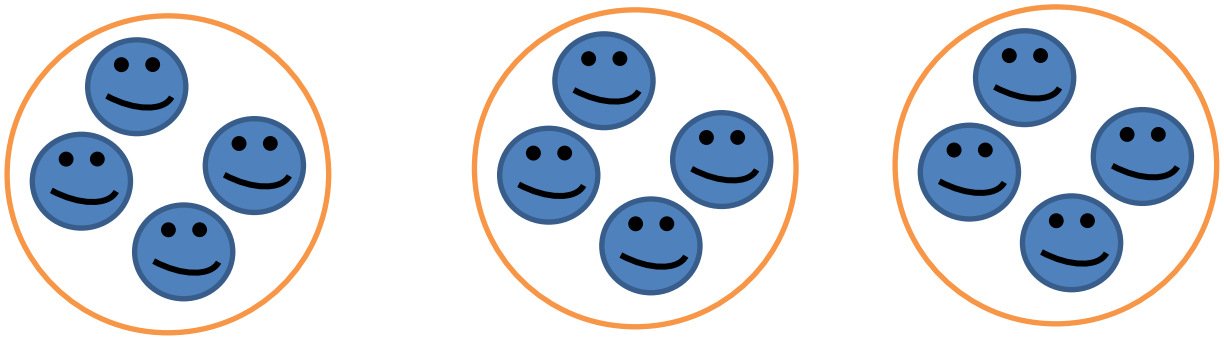
By the end of Year 6, children will have a range of calculation methods, mental and written.
Selection will depend upon the numbers involved.
Children should not go onto the next stage if:
1. they are not ready.
2. they are not confident.
Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to check their answers after calculation using an appropriate strategy.

PROGRESSION THROUGH CALCULATION FOR DIVISION

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. ***The mental calculation strategies taught will continue to be used and developed and should not be replaced by written methods.*** The following stages are standards that we expect the majority of our children to achieve.

Step 1

Children start by sharing objects (given in context) into equal groups. For example: 'I have 9 sweets to share between 3 people. How many sweets does each person get?'. Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

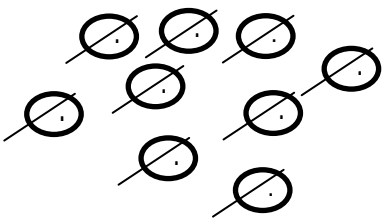


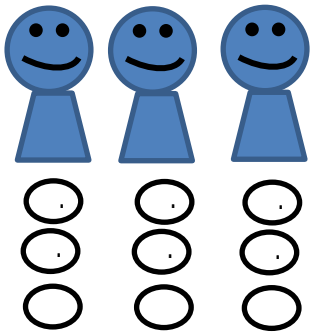
Step 2: Sharing pictorially (using pictures and then symbols)

Children will develop their understanding of division and use jottings to support calculation:

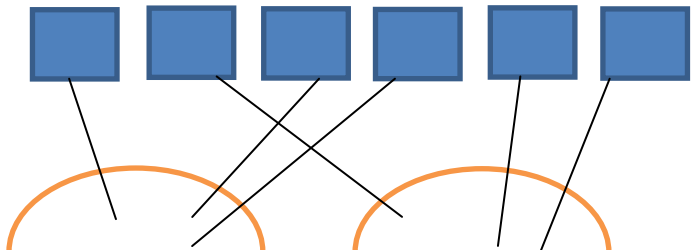
- Sharing Equally

9 ÷ 3 =

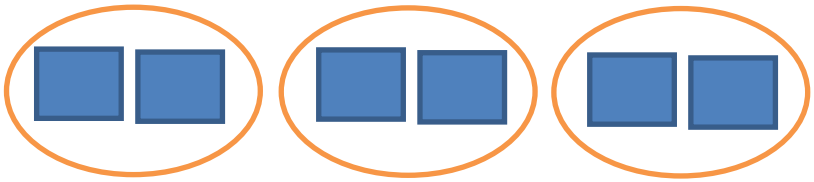




6 sweets shared between 2 people, how many do they get each?

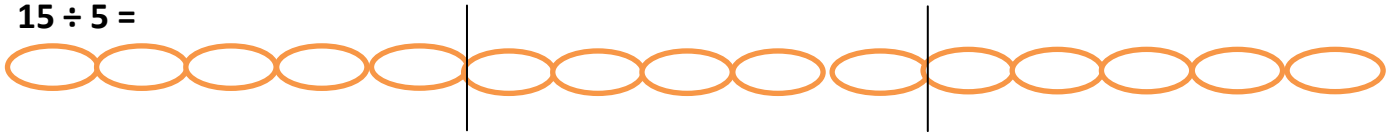


There are 6 sweets, how many people can have 2 sweets each?



Step 3: Linear grouping

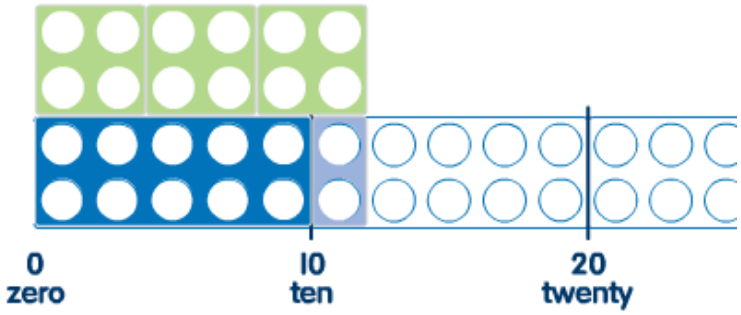
15 ÷ 5 =



Repeated addition to work out division using Numicon

Children start by making the first number with Numicon. They know they are finding out how many groups of 4 are in 12. They should place groups of 4 above the 12 to find out how many groups are needed.

12 ÷ 4 =



Alongside practically working out division children need to be taught to use repeated addition to solve.

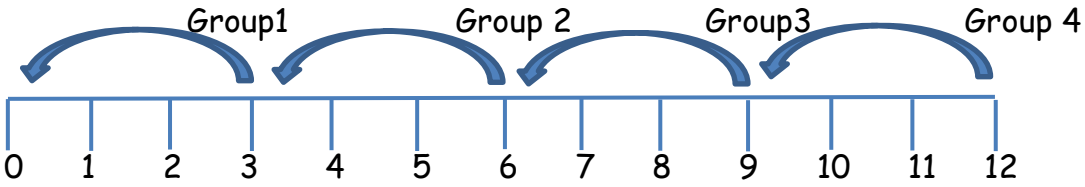
Explain how 12 is the total number, and the divisor is how many we are putting into each group. Model physically with Numicon, what are we counting in groups of? How do you know? When do I need to stop counting in 4's?

What is my repeated addition number sentence?
4 + 4 + 4 =12
How many groups of 4 have I got? 12 ÷ 4 = 3

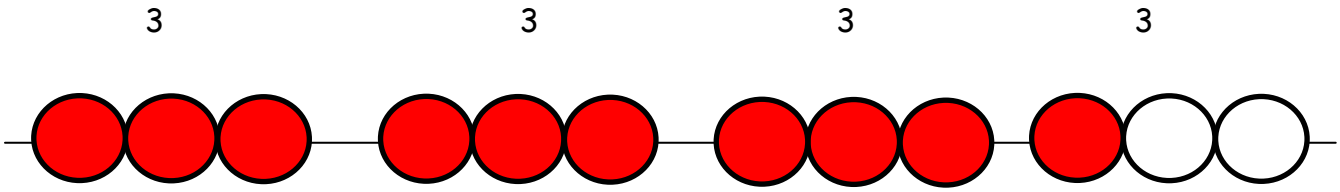
Step 4: Dividing using a number line – Phase B and C

Repeated subtraction using a numbered line or bead string or bead bar.

12 ÷ 3 = 4

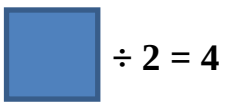


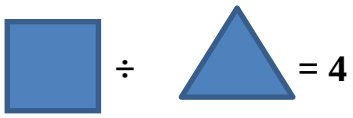
When children are secure with practical methods move onto drawing a number line to workout division. Children will still need to be working this out practically first so they can see how the number line represents the same process.



The bead string can help children with interpreting division calculations such as 10 ÷ 3 as
'how many 3s make 10?'

- Using symbols to stand for unknown numbers to complete equations using inverse operations.

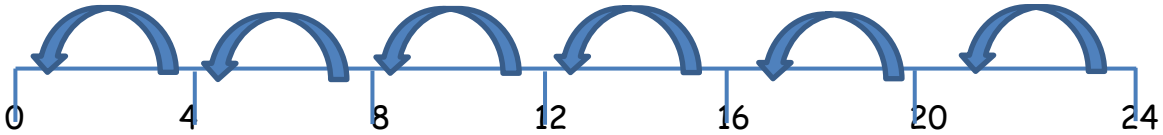




Ensure that a lot of emphasis from this point onwards is on grouping rather than sharing.
Children in Phase B will continue to use:

- Repeated subtraction using a number line

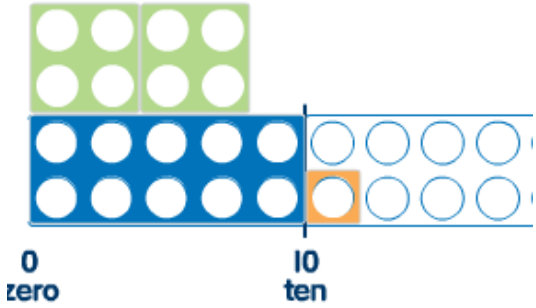
24 ÷ 4 = 6



Step 5: Dividing TU ÷ U with remainders

Children will need to use Numicon to before completing on a number line.

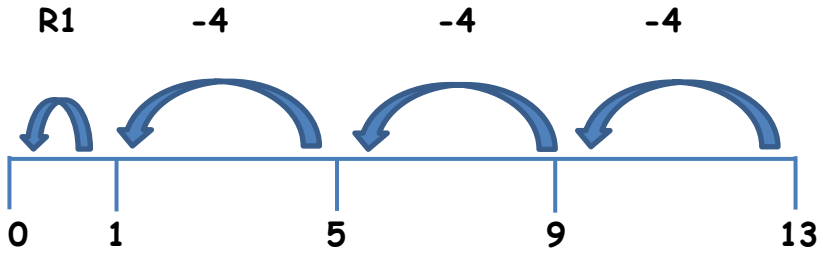
11 ÷ 4 =



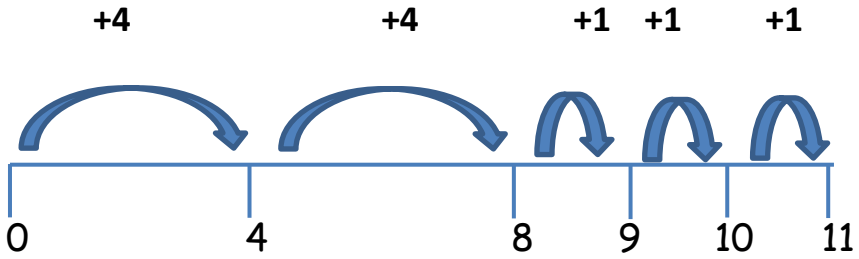
Build the number (11) first what am I counting in groups of? (4) Can I place on a 3rd group of 4? How many whole groups do I have? What do I call the leftovers? Model how to write 2 r 3.

- Children should move onto calculations involving remainders using a number line

13 ÷ 4 = 3 r 1



11 ÷ 4 =



2 groups of 4 and 3 left over = 2 remainder 3

Extend to higher 2 digit numbers.

Step 6: Vertical Chunking/Expanded vertical method

- Short division TU ÷ U

72 ÷ 3

```
3 | 72
  - 30      3 x 10
  ----
    42
  - 30      3 x 10
  ----
    12
  -  6      3 x 2
```

Ready table: What do I already know?

1	3
2	6
4	12
8	24
10	30

(Children should create and use this table to help find chunks.

	6		
-	6	3 x (2)	Answer: 24
	0		

96 ÷ 5 = 19 r 1

5 ⟌	96		
-	50	5 x (10)	
	46		
-	30	5 x (6)	
	16		
-	15	5 x (3)	
	1		

Children use their knowledge of multiples of 5 to reduce down to zero or the remainder.

Children need to 'bank' how many groups of 5 they have found. Then add the 'banked' numbers together (10+6+3=19).

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.
Children need to be able to decide what to do after division and round up and down accordingly – this will depend on the context of the problem.

For example:
I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet).
Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
Answer: 8 (the remaining 6 apples still need to be placed into a box).
Children will continue to use written methods to solve short division TU ÷ U.
Children need to start to subtract larger multiples of the divisor.

Step 7: TU ÷ TU using expanded vertical method (Chunking)

- Short division (Chunking) HTU ÷ U

196 ÷ 6

6 ⟌	196		
-	180	6 x 30	
	16		
-	12	6 x 2	
	4		Answer: 32 r 4

- Long Division (Chunking) HTU ÷ TU

972 ÷ 36 = 27

	27		
36 ⟌	972		
-	720	36 x 20	
	252		
-	180	36 x 5	
	72		
-	72	36 x 2	

0

Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10; the answer should be shown as 3 2/10 which could be written as 3 1/5 in its simplest term.

- Using decimals (Chunking)

87.5 ÷ 7

```
      12.5
   ┌──────
 7 │ 87.5
 - │ 70.0    7 x 10
   ──────
     17.5
 - │ 14.0    7 x 2
   ──────
      3.5
 - │  3.5    7 x 0.5
   ──────
         0
```

Children need to be able to decide what to do after division and round up and down accordingly – this will depend on the context of the problem.

Step 8: Compact Vertical Method (Short formal division)

Also known as the Guzinter method or bus stop method when dividing by a 1 digit number. It gets its name from the language used: ' 7 *goes into* 62 eight times remainder 6'. This can be a quicker method to use especially for decimals. It is a difficult method to understand mathematically, and will not be taught until children in Phase C are familiar with the repeated subtraction model.

- Short Division TU ÷ U

72 ÷ 3

```
    24
  ┌──
3 │ 72
```

- Short Division TU ÷ U with remainders

71 ÷ 4

```
    17  r 3
  ┌──────
4 │ 7³1
```

Further challenge would include writing the remainder as a fraction 3/4 or as a decimal 0.75.

71 ÷ 4 = 17.75

Children should:

- Be able to divide numbers with at least HTU ÷ U

For Example:

291 ÷ 3

```
    97
  ┌──────
3 │ 2²9²1
```

- Use this method to divide decimals

For Example:

87.5 ÷ 7

```
    12.5
  ┌──────
7 │ 8 ¹7. ³5
```

67.5 ÷ 6

$$\begin{array}{r} 11.25 \\ 6 \overline{)\,67.^{1}5^{3}0} \end{array}$$

As we do not know our times tables for 15, 19, 23 (and so on) wen tend to use the chunking method when dividing by a 2 digit number.

By the end of Year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not go onto the next stage if:

1. they are not ready.

2. they are not confident.

Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to check their answers after calculation using an appropriate strategy.

We hope this booklet is useful to support learning at home.
These written methods build on the vital mental calculation methods taught in the earlier years. Children need to continue to develop their mental calculation skills after they start using the written methods. If introduced too early, written methods can hold back children's mathematical understanding. Even once learnt, children are encouraged to look at each calculation and choose the best method to solve it with.
If you have any questions regarding the different methods please contact your child's class teacher.

This booklet has been created to show you the stages your child will through when learning how to add, subtract, multiply and divide.
Some children will miss out some stages while others may use additional strategies to consolidate their understanding. Children will only be moved in to the next stage when ready.
Children are not expected to reach some of the later strategies until Upper Key Stage 2 (years 5 & 6).
By the end of Year 6, all children will have a range of methods, both mental and written to use when solving calculations involving all four of the operations.

Children should be encouraged to:-

- Approximate their answers before calculating
- Check their answers after calculating
- Consider if a mental method would be appropriate before using a written one.