Upper key stage 2 – years 5 and 6

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Year 5 programme of study

Number – number and place value

Statutory requirements

Pupils should be taught to:

read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit

count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000

interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000

solve number problems and practical problems that involve all of the above

read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

Notes and guidance (non-statutory)

Pupils identify the place value in large whole numbers.

They continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions that they have met so far.

They should recognise and describe linear number sequences, including those involving fractions and decimals, and find the term-to-term rule.

They should recognise and describe linear number sequences (for example, 3, $3\frac{1}{2}$, 4, 4 $\frac{1}{2}$...), including those involving fractions and decimals, and find the term-to-term rule in words (for example, add $\frac{1}{2}$).

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

add and subtract numbers mentally with increasingly large numbers

use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1).

They practise mental calculations with increasingly large numbers to aid fluency (for example, 12 462 – 2300 = 10 162).

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers

establish whether a number up to 100 is prime and recall prime numbers up to 19

multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

multiply and divide numbers mentally drawing upon known facts

divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Statutory requirements

multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})

solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division (see Mathematics Appendix 1). They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).

Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.

Distributivity can be expressed as $a(b + c) = ab + ac$.

They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10).

Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, 13 + 24 = 12 + 25; 33 = 5 x □).

Number – fractions (including decimals and percentages)

Statutory requirements

Pupils should be taught to:

compare and order fractions whose denominators are all multiples of the same number

identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$]

add and subtract fractions with the same denominator and denominators that are multiples of the same number

multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

read and write decimal numbers as fractions [for example, $0.71 = \frac{71}{100}$]

recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents

round decimals with two decimal places to the nearest whole number and to one decimal place

read, write, order and compare numbers with up to three decimal places

solve problems involving number up to three decimal places

recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal

solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

Notes and guidance (non-statutory)

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.

They extend their knowledge of fractions to thousandths and connect to decimals and measures.

Notes and guidance (non-statutory)

Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions > 1 to division with remainders, using the number line and other models, and hence move from these to improper and mixed fractions.

Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1.

Pupils practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They extend their understanding of adding and subtracting fractions to calculations that exceed 1 as a mixed number.

Pupils continue to practise counting forwards and backwards in simple fractions.

Pupils continue to develop their understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities.

Pupils extend counting from year 4, using decimals and fractions including bridging zero, for example on a number line.

Pupils say, read and write decimal fractions and related tenths, hundredths and thousandths accurately and are confident in checking the reasonableness of their answers to problems.

They mentally add and subtract tenths, and one-digit whole numbers and tenths.

They practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1).

Pupils should go beyond the measurement and money models of decimals, for example, by solving puzzles involving decimals.

Pupils should make connections between percentages, fractions and decimals (for example, 100% represents a whole quantity and 1% is $\frac{1}{100}$, 50% is $\frac{50}{100}$, 25% is $\frac{25}{100}$) and relate this to finding 'fractions of'.

Measurement

Statutory requirements

Pupils should be taught to:

convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)

understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints

measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres

calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes

estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]

solve problems involving converting between units of time

use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.

Notes and guidance (non-statutory)

Pupils use their knowledge of place value and multiplication and division to convert between standard units.

Pupils calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths. Missing measures questions such as these can be expressed algebraically, for example $4 + 2b = 20$ for a rectangle of sides 2 cm and b cm and perimeter of 20cm.

Pupils calculate the area from scale drawings using given measurements.

Pupils use all four operations in problems involving time and money, including conversions (for example, days to weeks, expressing the answer as weeks and days).

Geometry – properties of shapes

Statutory requirements

Pupils should be taught to:

identify 3-D shapes, including cubes and other cuboids, from 2-D representations

know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles

draw given angles, and measure them in degrees (^{o})

identify:

> angles at a point and one whole turn (total 360°)
>
> angles at a point on a straight line and $\frac{1}{2}$ a turn (total 180°)
>
> other multiples of 90°

use the properties of rectangles to deduce related facts and find missing lengths and angles

distinguish between regular and irregular polygons based on reasoning about equal sides and angles.

Notes and guidance (non-statutory)

Pupils become accurate in drawing lines with a ruler to the nearest millimetre, and measuring with a protractor. They use conventional markings for parallel lines and right angles.

Pupils use the term diagonal and make conjectures about the angles formed between sides, and between diagonals and parallel sides, and other properties of quadrilaterals, for example using dynamic geometry ICT tools.

Pupils use angle sum facts and other properties to make deductions about missing angles and relate these to missing number problems.

Geometry – position and direction

Statutory requirements

Pupils should be taught to:

identify, describe and represent the position of a shape following a reflection or

Statutory requirements

translation, using the appropriate language, and know that the shape has not changed.

Notes and guidance (non-statutory)

Pupils recognise and use reflection and translation in a variety of diagrams, including continuing to use a 2-D grid and coordinates in the first quadrant. Reflection should be in lines that are parallel to the axes.

Statistics

Statutory requirements

Pupils should be taught to:

solve comparison, sum and difference problems using information presented in a line graph

complete, read and interpret information in tables, including timetables.

Notes and guidance (non-statutory)

Pupils connect their work on coordinates and scales to their interpretation of time graphs.

They begin to decide which representations of data are most appropriate and why.

Year 6 programme of study

Number – number and place value

Statutory requirements

Pupils should be taught to:

read, write, order and compare numbers up to 10 000 000 and determine the value of each digit

round any whole number to a required degree of accuracy

use negative numbers in context, and calculate intervals across zero

solve number and practical problems that involve all of the above.

Notes and guidance (non-statutory)

Pupils use the whole number system, including saying, reading and writing numbers accurately.

Number – addition, subtraction, multiplication and division

Statutory requirements

Pupils should be taught to:

multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

perform mental calculations, including with mixed operations and large numbers

identify common factors, common multiples and prime numbers

use their knowledge of the order of operations to carry out calculations involving the four operations

Statutory requirements
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

solve problems involving addition, subtraction, multiplication and division
use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)
Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1).
They undertake mental calculations with increasingly large numbers and more complex calculations.
Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.
Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.
Common factors can be related to finding equivalent fractions.

Number – fractions (including decimals and percentages)

Statutory requirements
Pupils should be taught to:
use common factors to simplify fractions; use common multiples to express fractions in the same denomination
compare and order fractions, including fractions > 1
add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]
divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]

associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]

identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places

multiply one-digit numbers with up to two decimal places by whole numbers

use written division methods in cases where the answer has up to two decimal places

solve problems which require answers to be rounded to specified degrees of accuracy

recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Notes and guidance (non-statutory)

Pupils should practise, use and understand the addition and subtraction of fractions with different denominators by identifying equivalent fractions with the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other (for example, $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$) and progress to varied and increasingly complex problems.

Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers, and as equal parts of objects, for example as parts of a rectangle.

Pupils use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (for example, if $\frac{1}{4}$ of a length is 36cm, then the whole length is 36 × 4 = 144cm).

They practise calculations with simple fractions and decimal fraction equivalents to aid fluency, including listing equivalent fractions to identify fractions with common denominators.

Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3 ÷ 8 = 0.375). For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to three decimal places, or other appropriate approximations depending on the context. Pupils multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers. Pupils multiply decimals by whole numbers, starting with the simplest cases, such as 0.4 × 2 = 0.8, and in practical contexts, such as measures and money.

Pupils are introduced to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money. They recognise division calculations as the inverse of multiplication.

Notes and guidance (non-statutory)

Pupils also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations. This includes rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.

Ratio and proportion

Statutory requirements

Pupils should be taught to:

solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

solve problems involving similar shapes where the scale factor is known or can be found

solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

Notes and guidance (non-statutory)

Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (for example, similar shapes and recipes).

Pupils link percentages or 360° to calculating angles of pie charts.

Pupils should consolidate their understanding of ratio when comparing quantities, sizes and scale drawings by solving a variety of problems. They might use the notation *a:b* to record their work.

Pupils solve problems involving unequal quantities, for example, 'for every egg you need three spoonfuls of flour', '$\frac{3}{5}$ of the class are boys'. These problems are the foundation for later formal approaches to ratio and proportion.

Algebra

Statutory requirements

Pupils should be taught to:

use simple formulae

generate and describe linear number sequences

express missing number problems algebraically

find pairs of numbers that satisfy an equation with two unknowns

enumerate possibilities of combinations of two variables.

Notes and guidance (non-statutory)

Pupils should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand, such as:

missing numbers, lengths, coordinates and angles

formulae in mathematics and science

equivalent expressions (for example, $a + b = b + a$)

generalisations of number patterns

number puzzles (for example, what two numbers can add up to).

Measurement

Statutory requirements

Pupils should be taught to:

solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate

use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places

convert between miles and kilometres

recognise that shapes with the same areas can have different perimeters and vice versa

recognise when it is possible to use formulae for area and volume of shapes

Statutory requirements
calculate the area of parallelograms and triangles
calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3].

Notes and guidance (non-statutory)
Pupils connect conversion (for example, from kilometres to miles) to a graphical representation as preparation for understanding linear/proportional graphs.
They know approximate conversions and are able to tell if an answer is sensible.
Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.
They relate the area of rectangles to parallelograms and triangles, for example, by dissection, and calculate their areas, understanding and using the formulae (in words or symbols) to do this.
Pupils could be introduced to compound units for speed, such as miles per hour, and apply their knowledge in science or other subjects as appropriate.

Geometry – properties of shapes

Statutory requirements
Pupils should be taught to:
draw 2-D shapes using given dimensions and angles
recognise, describe and build simple 3-D shapes, including making nets
compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

Notes and guidance (non-statutory)
Pupils draw shapes and nets accurately, using measuring tools and conventional

markings and labels for lines and angles.

Pupils describe the properties of shapes and explain how unknown angles and lengths can be derived from known measurements.

These relationships might be expressed algebraically for example, $d = 2 \times r$; $a = 180 - (b + c)$.

Geometry – position and direction

Statutory requirements

Pupils should be taught to:

describe positions on the full coordinate grid (all four quadrants)

draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

Notes and guidance (non-statutory)

Pupils draw and label a pair of axes in all four quadrants with equal scaling. This extends their knowledge of one quadrant to all four quadrants, including the use of negative numbers.

Pupils draw and label rectangles (including squares), parallelograms and rhombuses, specified by coordinates in the four quadrants, predicting missing coordinates using the properties of shapes. These might be expressed algebraically for example, translating vertex (a, b) to $(a – 2, b + 3)$; (a, b) and $(a + d, b + d)$ being opposite vertices of a square of side d.

Statistics

Statutory requirements

Pupils should be taught to:

interpret and construct pie charts and line graphs and use these to solve problems

calculate and interpret the mean as an average.

Notes and guidance (non-statutory)

Pupils connect their work on angles, fractions and percentages to the interpretation of pie charts.

Pupils both encounter and draw graphs relating two variables, arising from their own enquiry and in other subjects.

They should connect conversion from kilometres to miles in measurement to its graphical representation.

Pupils know when it is appropriate to find the mean of a data set.

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes	874 − 523 becomes	932 − 457 becomes	932 − 457 becomes
$$\begin{array}{r} 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \\ {\scriptstyle 1\ \ 1} \end{array}$$	$$\begin{array}{r} 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \end{array}$$	$$\begin{array}{r} {\scriptstyle 8\ \ 12\ 1} \\ 9\!\!\!/\ 3\!\!\!/\ 2 \\ -\ 4\ 5\ 7 \\ \hline 4\ 7\ 5 \end{array}$$	$$\begin{array}{r} {\scriptstyle 1\ \ \ 1} \\ 9\ 3\ 2 \\ -\ 4\!\!\!/\ 5\!\!\!/\ 7 \\ {\scriptstyle 5\ \ 6} \\ \hline 4\ 7\ 5 \end{array}$$
Answer: 1431	Answer: 351	Answer: 475	Answer: 475

Short multiplication

24 × 6 becomes	342 × 7 becomes	2741 × 6 becomes
$$\begin{array}{r} 2\ 4 \\ \times\ \ \ 6 \\ \hline 1\ 4\ 4 \\ {\scriptstyle 2} \end{array}$$	$$\begin{array}{r} 3\ 4\ 2 \\ \times\ \ \ \ \ 7 \\ \hline 2\ 3\ 9\ 4 \\ {\scriptstyle 2\ 1} \end{array}$$	$$\begin{array}{r} 2\ 7\ 4\ 1 \\ \times\ \ \ \ \ \ \ 6 \\ \hline 1\ 6\ 4\ 4\ 6 \\ {\scriptstyle 4\ 2} \end{array}$$
Answer: 144	Answer: 2394	Answer: 16 446

Long multiplication

24 × 16 becomes

```
        2
    2   4
×   1   6
─────────
2   4   0
1   4   4
─────────
3   8   4
─────────
```

Answer: 384

124 × 26 becomes

```
    1   2
1   2   4
×       2   6
─────────────
2   4   8   0
    7   4   4
─────────────
3   2   2   4
    1   1
```

Answer: 3224

124 × 26 becomes

```
    1   2
1   2   4
×       2   6
─────────────
    7   4   4
2   4   8   0
─────────────
3   2   2   4
    1   1
```

Answer: 3224

Short division

98 ÷ 7 becomes

```
        1   4
      ┌─────────
      │   2
    7 │ 9   8
```

Answer: 14

432 ÷ 5 becomes

```
        8   6   r 2
      ┌─────────
      │       3
    5 │ 4   3   2
```

Answer: 86 remainder 2

496 ÷ 11 becomes

```
          4   5   r 1
        ┌─────────
        │       5
  1   1 │ 4   9   6
```

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

```
            2   8   r 12
        ┌─────────
  1   5 │ 4   3   2
          3   0   0
        ─────────
          1   3   2
          1   2   0
        ─────────
              1   2
```

Answer: 28 remainder 12

432 ÷ 15 becomes

```
            2   8
        ┌─────────
  1   5 │ 4   3   2
          3   0   0      15×20
        ─────────
          1   3   2
          1   2   0      15×8
        ─────────
              1   2
```

$$\frac{12}{15} = \frac{4}{5}$$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

```
            2   8 · 8
        ┌─────────────
  1   5 │ 4   3   2 · 0
          3   0   ↓
        ─────────
          1   3   2
          1   2   0   ↓
        ─────────
              1   2   0
              1   2   0
            ─────────
                      0
```

Answer: 28·8