



NUMBERS - COUNTING

Numbers seem simple but actually are exceedingly complex. There are three different aspects, each of which is highly relevant to how we teach young children.

1. Numbers as names

When we give buses a number, we are effectively naming them. The number 3 bus is not called '3' because it is third in a line or because it goes to three places. Here, numbers are being used as names. House numbers, car numbers and many others are identifiers, in the same way as names are. We could call our cars Fred, Annie, Parvati, Sunil, Harry, etc. but we choose to use numbers instead.

Chanting the numbers in order, first to 20 and then on to 100, is an essential pre-requisite for all arithmetic. Great emphasis should be laid on the ability to count to 100 in company with the rest of the class, as without this skill, children's capacity to manipulate numbers is likely to suffer. We help children to recognise the numbers, starting with those up to 10 and 20, distinguish each numeral and identify numbered houses, etc. especially their own.

2. Numbers as the 'how-many-ness' of a set (Cardinality)

Children come to understand that we can count to establish how many are in a set. We find out how many objects, images, sounds or actions by counting.

- (i) One-to-one correspondence – children need to practise counting when they touch or move each object. This ensures that they match a spoken number to an object each time. The correct number in the set is established by the final number in the count.
- (ii) Conservation of number – children need to realise that number does not depend on arrangement. So if we count the number of cups on a tray and establish that there are six, then when we move the cups around the tray children should see that there is no need to re-count them; there are still six cups.
- (iii) Children understand zero (0) as the empty set, and also begin to understand that it can mean 'no 1s' in numbers such as 20 and 30.

3. Numbers as places in a line or sequence (Ordinality)

Seeing numbers pegged on a washing line, or as tiles making up a number track along the floor, allows children to appreciate the ordinal aspect of numbers. Understanding that numbers form a sequence and knowing that '4' comes between '3' and '5' on the number line, are as important as being able to count four things.

The number line (numbers represented visually in an ordered sequence) is arguably the most important model that we give primary

children. On their mathematical journey, the image of the number line will be developed through the use of the bead-bar and beaded line, the landmarked line and on to the use of empty number lines, and zoomed sections. So the pegged number line is an essential part of every nursery and reception classroom, and children's attention should be drawn to it on a daily basis. *Which number has gone walkabout? Which two numbers has 'naughty puppet' muddled up?*

All the above forms of counting are important. Children are taught to:

- Chant the numbers in order so that they can 'count' from 1 to 20 with total confidence and many can count from 1 to 100
- Establish how many there are in a set by counting, making one-to-one correspondence, and understanding conservation of number
- Identify and place numbers on a line, compare numbers saying which is larger or smaller and order a set of numbers.

Positive Relationships: what adults can do

30-40 months	40-60 months
<ul style="list-style-type: none"> • Use number language, e.g. 'one', 'two', 'three', 'lots', 'fewer', 'hundreds', 'how many?' and 'count' in a variety of situations. • Support children's developing understanding of abstraction by counting things that are not objects, such as hops, jumps, clicks or claps. • Model counting of objects in a random layout, showing the result is always the same as long as each object is only counted once. • Model and encourage use of mathematical language, e.g. asking questions such as, 'How many saucepans will fit on the shelf?' • Use pictures and objects to illustrate counting songs, rhymes and number stories. 	<ul style="list-style-type: none"> • Encourage estimation, e.g. how many sandwiches are needed for the picnic. • Encourage use of mathematical language, e.g. number names to ten: 'Have you got enough to give me three?' • Add numerals to all areas of learning and development, e.g. to a display of a favourite story, such as <i>The Three Billy Goats Gruff</i>. • Emphasise the empty set and introduce the concept of nothing or zero. • Make books about numbers that have meaning for the child such as favourite numbers, birth dates or telephone numbers. • Show interest in how children solve problems and value their different solutions. • Make sure children are secure about the order of numbers before asking what comes after or before each number. • Use mathematical vocabulary and demonstrate methods of recording, using standard notation where appropriate. • Give children who are learning English as additional language opportunities to work in their home language, to ensure accurate understanding of concepts.

NUMBERS – ADDITION AND SUBTRACTION

Addition has two aspects and both need to be developed in parallel.

1. Adding as counting on

Children need to understand that adding involves increase. Thus, although children may combine two sets by counting all of the objects, we need to progress to adding two sets by starting with the larger and counting on by the smaller, e.g. we start with five ducks on the pond and add three ducks by counting each one as they swim in – start with five, then six, seven, eight. We now have eight.

The essential pre-requisite for addition is being able to say the 'next number' *without counting from one*. So, if we say 'six', it is crucial that the child can respond 'seven' without having to begin at one and count up. Saying the number one more than any number up to first 10, and then 20, is a key skill for Reception children as their subsequent work in addition hangs on it.

We start recording additions using a number sentence with + and =. It is exceptionally important that children encounter and, with the teacher and peers, read these sentences aloud, and it is less important that they are able to write them. Laying out addition sentences using magnetic numbers/symbols on whiteboards is very useful and helps avoid the need for good hand-to-eye coordination. Lack of this could hold up mathematical development.

Once children can say the next number, they can count on small amounts. Additions such as $5 + 2$ and $8 + 3$ become possible, using fingers as markers in the count. We start with 8, putting it in our heads, and count on using one finger for each number spoken: 9, 10, 11.

2. Adding as splitting sets: number bonds

Knowing that 5 can be split into 4 and 1, into 3 and 2 and even into 5 and 0, is the basis for this aspect of addition. We do not want children to have to count on to add 3 and 2 more, any more than we want them to count five things; they should subitise, i.e. recognise that there are five without counting (see Number). Subitising is the basis for number bonds.

Teaching children that 5 can be split into $5 + 0$, $4 + 1$ and $3 + 2$, that 6 can be split into $6 + 0$, $5 + 1$, $4 + 2$ and $3 + 3$, etc. is essential in Reception. Using strings of four, five or six beads, which children can fiddle with, four, five or six pegs on a hanger and four, five or six cubes in a tower, means that children can play with these sets and kinaesthetically absorb the relevant number bonds (pairs of numbers that make 4, 5 and 6), and begin to learn the bonds for 7, 8 and 9.

Children need to see adults recording these additions using number sentences, preferably arranged so as to make the patterns clear:

$$0 + 5 = 5$$

$$1 + 4 = 5$$

$$2 + 3 = 5$$

Bonds to 10 are essential for the development of numerical fluency, and these are started in Reception. Children at this early stage are highly receptive and will absorb these crucial pairs. If the number bonds for ten are chanted, modelled on fingers, with pegs on hangers and

using bead strings, on a weekly basis, there will not be a 5-year old child who does not know them off by heart – a really good predictor for later fluency.

Subtraction has three aspects. Two of these should be developed in parallel, and one is touched on in Reception, but not really developed until Y1.

1. Subtraction as counting back or taking away

This is perhaps the most intuitive aspect of subtraction. Children can establish how many in a set and then count back as one, two or three objects are removed. This is recorded using the '-' sign. There are six apricots on a plate, we eat two, how many are left?

Teacher records as $6 - 2 = 4$ and reads this with the children as six take away two leaves four or six subtract two equals four. It is important to read the - sign as 'take away', and also as 'count back' and as 'subtract'.

As children progress, they are introduced to the notion that we can model this 'taking away' on the number line. We point at six and count back two to reach four. $6 - 2 = 4$. Tiger Ted is on 9 on our number track. If we count on two, we reach 11. This models adding two more. If we count back two, we reach 7. This models taking away two.

2. Subtraction as number bonds

Just as addition must be related to splitting a number into two sets, 5 is 2 and 3 or 4 and 1, so subtraction needs to be seen as the inverse of this.

$$5 = 3 + 2$$

$$5 - 2 = 3$$

The easiest and quickest way of showing this is on fingers. Show five fingers, two on one hand and three on the other. Say the addition together. Three add two is five. Fold down the two fingers. Say the subtraction together. Five subtract two is three. Repeat, holding up all five fingers and saying the addition again. Then fold down three fingers and say the subtraction. Five subtract three is two.

3. Subtraction as difference

This aspect of subtraction will prove to be the most important for achieving numerical fluency by Year 3, when children will perform many subtractions by counting up. However, in Reception, we do not expect children to use counting up to subtract, even at a simple level.

However, we do want to draw upon the notion of difference since this is a very familiar concept for small children and one which resonates with their own experience. If Tyler has five strawberries and Cindy has three, it will be clear to both children that Tyler has more. And they will often be able to tell you how many more. So we draw upon children's experience of difference and use the language of 'how many more?' and 'what is the difference?' as we record these situations.

Tyler has 5; Cindy has 3; Tyler has two more than Cindy.

It is the vocabulary of difference that we are developing here, and this is especially helpful in the context of measures. The pencil is six cubes long. The crayon is four cubes long. The pencil is two cubes longer. However, this is not recorded as a formal subtraction at this early stage.

Positive Relationships: what adults can do	
30-40 months	40-60 months
<ul style="list-style-type: none"> • Use number language, e.g. 'one', 'two', 'three', 'lots', 'fewer', 'hundreds', 'how many?' and 'count' in a variety of situations. • Model and encourage use of mathematical language, e.g. asking questions such as 'How many saucepans will fit on the shelf?' • As you read number stories or rhymes, ask e.g. '<i>When one more frog jumps in, how many will there be in the pool altogether?</i>' • Use pictures and objects to illustrate counting songs, rhymes and number stories. • Encourage children to use mark-making to support their thinking about numbers and simple problems. • Talk with children about the strategies they are using, e.g. to work out a solution to a simple problem by using fingers or counting aloud. 	<ul style="list-style-type: none"> • Encourage estimation e.g. how many sandwiches are needed for the picnic. • Encourage use of mathematical language, e.g. number names to ten: '<i>Have you got enough to give me three?</i>' • Ensure that children are involved in making displays, e.g. making their own pictograms of lunch choices. Develop this as a 3D representation using bricks and discuss the most popular choices. • Use rhymes, songs and stories involving counting on and counting back in 1s, 2s, 5s and 10s. • Emphasise the empty set and introduce the concept of nothing or zero. • Show interest in how children solve problems and value their different solutions. • Discuss with children how problems relate to others they have met, and their different solutions. • Talk about the methods children use to answer a problem they have posed, e.g. '<i>Get one more, and then we will both have two.</i>' • Encourage children to make up story problems for other children to solve. • Use mathematical vocabulary and demonstrate methods of recording, using standard notation where appropriate. • Give children who are learning English as additional language opportunities to work in their home language, to ensure accurate understanding of concepts.

NUMBERS – MULTIPLICATION AND DIVISION

Multiplication is introduced in Reception as 'clever counting' and also as doubling.

Division relates to halving and sharing into two or four sets.

1. Clever Counting

Once children are confident at counting from 1 to 20 individually, and from 1 to 100 as part of a larger group, they will start counting in 2s, 5s and 10s.

Counting in 2s helps children to develop the concept of even and odd numbers, and is also an essential pre-requisite for doubling and halving. We like children to start by doing 'whisper' counting where they say every number, but the even numbers are whispered. One, two, three, four, five, six... etc. They can then progress to just saying the whispered numbers: two, four, six, etc. This chant, in 2s, to 20 is learned by heart.

Counting in 5s comes a little later and is, again, an easy chant to learn. We match this to hands, showing that five, ten, fifteen, twenty can be shown by holding up one, two, three, four hands.

Counting in 10s is not only useful as a precursor for the $10\times$ table, but really helps children to distinguish the -ty numbers from the -teen numbers, e.g. thirty from thir**teen**. We call the multiples of ten the 'cuppa tea' numbers to emphasis the ending.

2. Doubling

Children can start by doubling the numbers 1 to 5 using the fingers of both hands. The 'one and one is two, two and two is four', etc. chant, where thumbs on both hands are held up, then finger and thumb, then three digits, etc. is an excellent way of helping children really learn these doubles. Creating doubles using cubes and beads and other mathematical equipment helps to reinforce the fact that a double is a number multiplied by 2; there are two 5s in double 5.

3. Division as sharing

Once children understand their doubles, it is a natural progression to create the matching halves. *Double two is four and half of four is two*. We can then demonstrate that some numbers of cubes can be halved (even numbers) and some can't (odd numbers). It is important to emphasise that if we are talking about cakes or biscuits, odd numbers can be halved, i.e. we can share three cakes fairly – we get $1\frac{1}{2}$ each! Children need to understand that we can find half of small odd numbers, as well as knowing the halves of the even numbers up to 10.

Once children are very familiar with the concept of halving, we can model sharing small multiples of four into quarters. So, for example, eight toy cars may be fairly shared amongst four children. Or a large cake may be cut into quarters.

It is important to emphasise that we are only expecting Reception children to get to grips with one aspect of division – that aspect of 'sharing' which is related to fractions. We are not demonstrating the arguably more important aspect of division as the inverse of multiplication – namely grouping. Four groups of five make twenty ($4 \times 5 = 20$). How many groups of five in 20? Children will be introduced to formal multiplication in Year 1 and division as grouping is taught mainly in Year 2.

Positive Relationships: what adults can do

30-40 months	40-60 months
<ul style="list-style-type: none"> • Model and encourage use of mathematical language, e.g. asking questions such as 'How many saucepans will fit on the shelf?' • Help children to understand that one thing can be shared by a number of pieces, e.g. a pizza. • Use pictures and objects to illustrate counting songs, rhymes and number stories. • Talk with children about the strategies they are using, e.g. to work out a solution to a simple problem by using fingers or counting aloud. 	<ul style="list-style-type: none"> • Encourage use of mathematical language, e.g. number names to ten: <i>'Have you got enough to give me three?'</i> • Ensure that children are involved in making displays, e.g. making their own pictograms of lunch choices. Develop this as a 3D representation using bricks and discuss the most popular choices. • Use rhymes, songs and stories involving counting on and counting back in 1s, 2s, 5s and 10s. • Discuss with children how problems relate to others they have met, and their different solutions. • Talk about the methods children use to answer a problem they have posed, e.g. <i>'Get one more, and then we will both have two.'</i> • Encourage children to make up their own story problems for other children to solve. • Encourage children to extend problems, e.g. <i>'Suppose there were three people to share the bricks between instead of two.'</i> • Use mathematical vocabulary and demonstrate methods of recording, using standard notation where appropriate. • Give children who are learning English as additional language opportunities to work in their home language, to ensure accurate understanding of concepts.

SHAPE, SPACE AND MEASURE

Shape

Children are introduced to a variety of 2D and 3D shapes. They explore their properties, and develop the use of specific vocabulary to describe these. Using words such as 'corners, sides, straight, curved, flat, roll, surface' as well as the names of common 2D and 3D shapes, children can identify, talk about and describe these in the world around them.

Children learn the difference between 2D and 3D shapes; they construct 3D shapes using a wide variety of construction materials, and then describe the 2D shapes they see on the faces of 3D shapes. They use these in activities such as printing.

A wide variety of shapes will be drawn upon, following children's own experiences, but in particular children will name and discuss rectangles, including squares and oblongs, circles, triangles, cubes, cuboids, spheres, cones and pyramids.

Children begin to appreciate symmetry, and create and identify symmetrical patterns.

Space

Children are introduced to the language of position and direction. They identify forward and backward, and begin to learn left and right. Words such as up, down, below, above, inside and outside start to become familiar and children are encouraged to describe where objects are in relation to others. They indicate direction, both by gesture and vocabulary, and are encouraged to use increasingly more specific terminology, such as behind, in front of, beside, next to.

Measure

Children begin to see that we can find out how long or wide or tall things are, either by comparing them directly, or by measuring them using uniform non-standard units. They will measure their own height in crayons and see that Jimmy is taller than they are because more crayons fit along his length. In this way, they can compare two or three items.

This progression, from direct comparison to the use of a non-standard uniform unit to measure and compare, is used in the development of children's understanding of mass and capacity. Children first compare directly, pouring a teapot into a jug and vice versa, and balancing a shoe against a small book. They then measure, weighing the shoe and the book in marbles, and counting how many cups of tea the teapot will pour for our dolls' picnic.

Children begin to understand that we can tell the time and that we speak of time in relation to days and nights (how many sleeps?) and in relation to longer time spans: weeks, months and years. They compare lengths of time, talking of longer and shorter time spans. They sequence familiar events, and begin to relate these to o'clock times on a daily schedule, understanding that they can measure short periods of time using egg timers, sun dials and water clocks as well as digital and analogue clocks.

Children learn to identify coins and begin to use these to create small amounts. Through playing shops and other experiences, including visiting real shops, children learn the processes of buying, paying and giving change. They become more proficient at coin recognition and are able to say which coins are higher value and will buy more.

Pattern

Children create and re-create patterns. They use a wide variety of objects, images and actions or sounds. They become proficient at pattern recognition and are able to use their skills of identifying patterns in relation to their work in number.

Positive Relationships: what adults can do	
30-40 months	40-60 months
<ul style="list-style-type: none"> • Demonstrate the language for shape, position and measures in discussions, e.g. '<i>sphere</i>', '<i>shape</i>', '<i>box</i>', '<i>in</i>', '<i>on</i>', '<i>inside</i>', '<i>under</i>', '<i>long</i>', '<i>longer</i>', '<i>longest</i>', '<i>short</i>', '<i>shorter</i>', '<i>shortest</i>', '<i>heavy</i>', '<i>light</i>', '<i>full</i>' and '<i>empty</i>'. • Find out and use equivalent terms for these in home languages. • Encourage children to talk about the shapes they see and use and how they are arranged and used in constructions. • Value children's constructions, e.g. helping to display them or taking photographs of them. 	<ul style="list-style-type: none"> • Ask 'silly' questions, e.g. show a tiny box and ask if there is a bicycle in it. • Play peek-a-boo, revealing shapes a little at a time and at different angles, asking children to say what they think the shape is, what else it could be or what it could not be. • Be a robot and ask children to give you instructions to get somewhere. Let them be the robot and you instruct them. • Introduce children to the use of mathematical names for 'solid' 3D shapes and 'flat' 2D shapes, and the mathematical terms to describe shapes. • Encourage children to use everyday words to describe position, e.g. when following pathways or playing with outdoor apparatus.