

# Calculation Pathway

## Addition

Year 1

Year 2

Year 3

### + = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$\begin{aligned} 2 &= 1 + 1 \\ 2 + 3 &= 4 + 1 \\ 3 &= 3 \\ 2 + 2 + 2 &= 4 + 2 \end{aligned}$$

Missing numbers need to be placed in all possible places.

$$\begin{array}{l} 3 + 4 = \quad \quad \quad = 3 + 4 \\ 3 + \quad = 7 \quad \quad \quad 7 = \quad + 4 \\ \quad + 4 = 7 \quad \quad \quad 7 = 3 + \quad \\ \quad + \nabla = 7 \quad \quad \quad 7 = \quad + \nabla \end{array}$$

### Activities

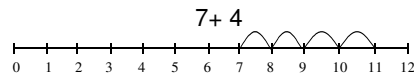
Children should have access to a wide range of counting equipment, everyday objects, as well as hoops, sorting trays, number tracks and numbered number lines.

### Teacher modelling

Drawing jumps on numbered number lines to support understanding of the mental method

### Children

To create their own jumps using rulers, fingers, pens, bodies etc.



### + = signs and missing numbers

Continue using a range of equations as in Year 1 but with appropriate, larger numbers. Extend to

$$\begin{aligned} 14 + 5 &= 10 + \quad \\ \text{and} \\ 32 + \quad + \quad &= 100 \quad 35 = 1 + \quad + 5 \end{aligned}$$

### Partition into tens and ones and recombine (using Dienes to see practically)

$$\begin{array}{r} 12 + 23 = \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ 10 \quad 2 \quad 20 \quad 3 \end{array}$$

$$\begin{aligned} 10 + 20 &= 30 \\ 2 + 3 &= 5 \\ 30 + 5 &= 35 \end{aligned}$$

### Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10

e.g.

Children should be able to partition the 7 to relate adding the 2 and then the 5.

$$8 + 7 = 15$$



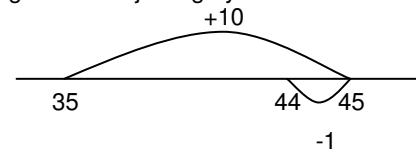
### Add 9 or 11 by adding 10 and adjusting by 1

Model using a number square and then develop to a number line.

e.g.

Add 9 by adding 10 and adjusting by 1

$$35 + 9 = 44$$



### + = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

### Partition into tens and ones

- Partition both numbers and recombine.
- Count on by partitioning the second number only e.g.

$$\begin{aligned} 36 + 53 &= 53 + 30 + 6 \\ &= 83 + 6 \\ &= 89 \end{aligned}$$

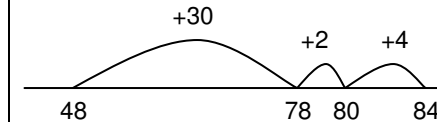
### Add a near multiple of 10 to a two-digit number

Secure mental methods by using a number line to model the method. Continue as in Year 2 but with appropriate numbers

e.g.  $35 + 19$  is the same as  $35 + 20 - 1$ .

Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.

$$48 + 36 = 84$$



### pencil and paper procedures

$$83 + 42 = 125$$

*either*

#### 1. Vertical expansion

$$\begin{array}{r} 83 \\ + 42 \\ \hline 5 \\ \hline 120 \\ \hline 125 \end{array}$$

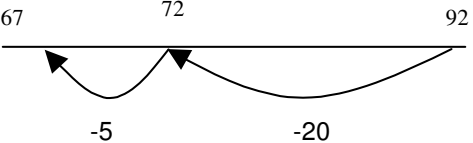
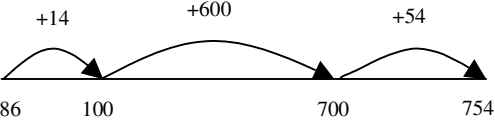
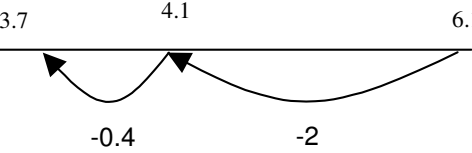
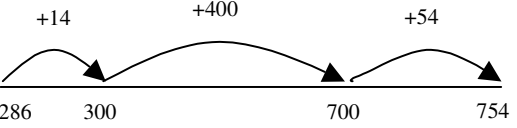
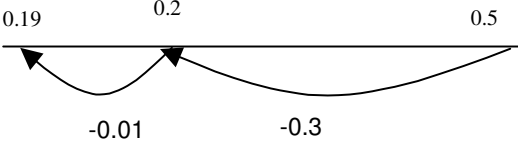
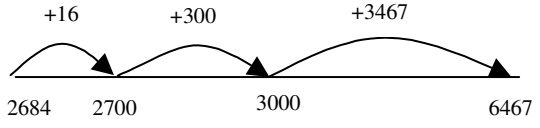
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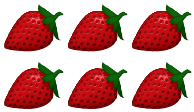
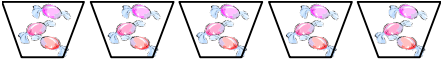
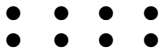
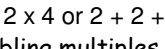

#### 2. Horizontal expansion

$$\begin{aligned} 80 + 3 \\ + 40 + 2 \\ \hline 120 + 5 = 125 \end{aligned}$$

	Addition																																	
Year 4	Year 5	Year 6																																
<p><b><u>+ = signs and missing numbers</u></b> Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.</p> <p><b><u>Partition into tens and ones and recombine</u></b> Either partition both numbers and recombine or partition the second number only e.g.  <math>55 + 37 = 55 + 30 + 7</math>  <math>= 85 + 7</math>  <math>= 92</math></p> <p><b><u>Add the nearest multiple of 10, then adjust</u></b></p> <p>Continue as in Year 2 and 3 but with appropriate numbers e.g. <math>63 + 29</math> is the same as <math>63 + 30 - 1</math></p> <p><b><u>Pencil and paper procedures</u></b>  <math>367 + 185 = 431</math>  <b>either</b>                      <b>or</b></p> <table style="display: inline-table; vertical-align: middle;"> <tr> <td style="text-align: right; padding-right: 10px;">367</td> <td style="text-align: left;">300 + 60 + 7</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">+185</td> <td style="text-align: left;">100 + 80 + 5</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">12</td> <td style="text-align: left;">400 + 140 + 12 = 552</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">140</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;"><u>400</u></td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">552</td> <td></td> </tr> </table> <p><b>leading to</b></p> <table style="display: inline-table; vertical-align: middle;"> <tr> <td style="text-align: right; padding-right: 10px;">367</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">+185</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;"><u>552</u></td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">11</td> <td></td> </tr> </table> <p>Extend to decimals in the context of money.</p>	367	300 + 60 + 7	+185	100 + 80 + 5	12	400 + 140 + 12 = 552	140		<u>400</u>		552		367		+185		<u>552</u>		11		<p><b><u>+ = signs and missing numbers</u></b> Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.</p> <p><b><u>Partition into hundreds, tens and ones and recombine</u></b> Either partition both numbers and recombine or partition the second number only e.g.  <math>358 + 73 = 358 + 70 + 3</math>  <math>= 428 + 3</math>  <math>= 431</math></p> <p><b><u>Add or subtract the nearest multiple of 10 or 100, then adjust</u></b> Continue as in Year 2, 3 and 4 but with appropriate numbers e.g. <math>458 + 79 =</math> is the same as <math>458 + 80 - 1</math></p> <p><b><u>Pencil and paper procedures</u></b> Extend to numbers with at least four digits  <math>3587 + 675 = 4262</math></p> <table style="margin-left: 40px;"> <tr><td style="text-align: right;">3587</td></tr> <tr><td style="text-align: right;">+ 675</td></tr> <tr><td style="text-align: right;"><u>4262</u></td></tr> <tr><td style="text-align: right;">111</td></tr> </table> <p>Revert to expanded methods if the children experience any difficulty. Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).</p> <table style="margin-left: 40px;"> <tr><td style="text-align: right;">72.8</td></tr> <tr><td style="text-align: right;">+54.6</td></tr> <tr><td style="text-align: right;"><u>127.4</u></td></tr> <tr><td style="text-align: right;">1 1</td></tr> </table>	3587	+ 675	<u>4262</u>	111	72.8	+54.6	<u>127.4</u>	1 1	<p><b><u>+ = signs and missing numbers</u></b> Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.</p> <p><b><u>Partition into hundreds, tens, ones and decimal fractions and recombine</u></b> Either partition both numbers and recombine or partition the second number only e.g.  <math>35.8 + 7.3 = 35.8 + 7 + 0.3</math>  <math>= 42.8 + 0.3</math>  <math>= 43.1</math></p> <p><b><u>Add the nearest multiple of 10, 100 or 1000, then adjust</u></b> Continue as in Year 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc</p> <p><b><u>Pencil and paper procedures</u></b> Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.  <math>13.86 + 9.481 = 23.341</math></p> <table style="margin-left: 40px;"> <tr><td style="text-align: right;">13.86</td></tr> <tr><td style="text-align: right;">+ 9.481</td></tr> <tr><td style="text-align: right;"><u>23.341</u></td></tr> <tr><td style="text-align: right;">1 1 1</td></tr> </table> <p>Revert to expanded methods if the children experience any difficulty.</p>	13.86	+ 9.481	<u>23.341</u>	1 1 1
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	Subtraction		
Year 4	Year 5	Year 6	
<p><b>- = signs and missing numbers</b> Continue using a range of equations as in Year 1 and 2 but with appropriate numbers. <b>Find a small difference by counting up</b> e.g. <math>5003 - 4996 = 7</math> This can be modelled on an empty number line (see complementary addition below). Children should be encouraged to use known number facts to reduce the number of steps. <b>Subtract the nearest multiple of 10, then adjust.</b> Continue as in Year 2 and 3 but with appropriate numbers. <b>Use known number facts and place value to subtract</b> <math>92 - 25 = 67</math></p>  <p style="text-align: right;">Pencil</p> <p>and paper procedures Complementary addition <math>754 - 86 = 668</math></p>  <p>For those children with a secure mental image of the number line they could record the jumps only: <math>754 - 86 = 668</math></p> $\begin{array}{r} 600 \\ 54 \\ + 14 \\ \hline 668 \end{array}$	<p><b>- = signs and missing numbers</b> Continue using a range of equations as in Year 1 and 2 but with appropriate numbers. <b>Find a difference by counting up</b> e.g. <math>8006 - 2993 = 5013</math> This can be modelled on an empty number line (see complementary addition below). <b>Subtract the nearest multiple of 10 or 100, then adjust.</b> Continue as in Year 2, 3 and 4 but with appropriate numbers. <b>Use known number facts and place value to subtract</b> <math>6.1 - 2.4 = 3.7</math></p>  <p style="text-align: right;">Pencil</p> <p>and paper procedures Complementary addition <math>754 - 286 = 468</math></p>  <p><b>Decomposition</b> <math>386 - 165 = 221</math> <math>300 + 80 + 6</math> <math>100 + 60 + 5</math> <math>200 + 20 + 1</math></p> <p><b>Standard written method</b> <math>386 - 165 = 221</math></p> $\begin{array}{r} 386 \\ - 165 \\ \hline 221 \end{array}$	<p><b>- = signs and missing numbers</b> Continue using a range of equations as in Year 1 and 2 but with appropriate numbers. <b>Find a difference by counting up</b> e.g. <math>8000 - 2785 = 5215</math> To make this method more efficient, the number of steps should be reduced to a minimum through children knowing:  <ul style="list-style-type: none"> <li>Complements to 1, involving decimals to two decimal places ( <math>0.16 + 0.84</math> )</li> <li>Complements to 10, 100 and 100</li> </ul> <b>Subtract the nearest multiple of 10, 100 or 1000, then adjust</b> Continue as in Year 2, 3, 4 and 5 but with appropriate numbers. <b>Use known number facts and place value to subtract</b> <math>0.5 - 0.31 =</math></p>  <p style="text-align: right;">Pencil</p> <p>and paper procedures Complementary addition <math>6467 - 2684 = 3783</math></p>  <p><b>Decomposition</b> <math>386 - 165 = 221</math> <math>300 + 80 + 6</math> <math>100 + 60 + 5</math> <math>200 + 20 + 1</math></p> <p><b>Standard written method</b> <math>386 - 165 = 221</math></p> $\begin{array}{r} 386 \\ - 165 \\ \hline 221 \end{array}$ <p>Extend to 2 places of decimals.</p>	

	Multiplication																	
Year 1	Year 2	Year 3																
<p>Multiplication is related to doubling and counting groups of the same size.</p>  <p>Looking at columns  <math>2 + 2 + 2</math>  3 groups of 2</p> <p>Looking at rows  <math>3 + 3</math>  2 groups of 3</p> <p><u>Counting using a variety of practical resources</u>  Counting in 2s e.g. counting socks, shoes, animal's legs...  Counting in 5s e.g. counting fingers, fingers in gloves, toes...  Counting in 10s e.g. fingers, toes...</p> <p><b>Pictures / marks</b></p> <p>There are 3 sweets in one bag.  How many sweets are there in 5 bags?</p> 	<p><u>Arrays and repeated addition</u></p>  $4 \times 2$ or $4 + 4$  $2 \times 4$ or $2 + 2 + 2 + 2$ <p><u>Doubling multiples of 5 up to 50</u>  <math>15 \times 2 = 30</math></p> <p>Partition</p> <p>Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: <math>6 = 5 + 1</math> so e.g. Double 6 is the same as double five add double one.</p>  <p>AND double 15</p> $\begin{array}{r} 10 + 5 \\ \downarrow \quad \downarrow \\ 20 + 10 = 30 \end{array}$ <p><u>x = signs and missing numbers</u></p> $7 \times 2 = \quad = 2 \times 7$ $7 \times \quad = 14$ $14 = \quad \times 7$ $\quad \times 2 = 14$ $14 = 2 \times \quad$ $\quad \times \nabla = 14$ $14 = \quad \times \nabla$	<p><u>x = signs and missing numbers</u>  Continue using a range of equations as in Year 2 but with appropriate numbers.</p> <p><u>Arrays and repeated addition</u>  Continue to understand multiplication as repeated addition and continue to use arrays (as in Year 2).</p> <p><u>Doubling multiples of 5 up to 50</u>  Doubles are learnt off by heart.  <math>35 \times 2 = 70</math></p> <p>Use known facts and place value to carry out simple multiplications, using partitioning.</p> <table border="1" data-bbox="1547 740 1868 831"> <tr> <td>X</td> <td>30</td> <td>5</td> <td></td> </tr> <tr> <td>2</td> <td>60</td> <td>10</td> <td>=70</td> </tr> </table> <p>By the Summer term children need to be applying the partitioning of digits in the grid method to begin multiplying using the refined formal short written method:</p> <p><math>13 \times 8 = 104</math></p> $\begin{array}{r} 13 \\ \times 8 \\ \hline 24 \quad (3 \times 8) \\ +80 \quad (10 \times 8) \\ \hline 104 \end{array}$ <p>They can then progress onto the short method without the expansion towards the end of the year if ready to.</p> <table data-bbox="1496 1273 1704 1370"> <tr> <td>13</td> <td>127</td> </tr> <tr> <td><math>\times 8</math></td> <td><math>\times 6</math></td> </tr> <tr> <td><math>\hline 104</math></td> <td><math>\hline 762</math></td> </tr> <tr> <td>2</td> <td>14</td> </tr> </table>	X	30	5		2	60	10	=70	13	127	$\times 8$	$\times 6$	$\hline 104$	$\hline 762$	2	14
X	30	5																
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2	14																	

Multiplication

Year 4	Year 5	Year 6
<p><b>Pencil and paper procedures</b> The refined version of the expanded short method to be continued from the end of year 3. Children to use this method with; <u>2d x 1d</u> (32 x 8) and <u>3d x 1 d</u> (142 x 7)</p> <p><b><u>13 x 8 = 104</u></b></p> $\begin{array}{r} 13 \\ \times 8 \\ \hline 24 \text{ (3 x 8)} \\ +80 \text{ (10 x 8)} \\ \hline 104 \end{array}$ <p><b><u>127 x 6 = 762</u></b></p> $\begin{array}{r} 127 \\ \times 6 \\ \hline 42 \text{ (6x7)} \\ 120 \text{ (6x20)} \\ +600 \text{ (6x100)} \\ \hline 762 \end{array}$ <p>The short method without the expansion should also be used.</p> $\begin{array}{r} 13 \\ \times 8 \\ \hline 104 \\ \hline 2 \end{array} \qquad \begin{array}{r} 127 \\ \times 6 \\ \hline 762 \\ \hline 14 \end{array}$	<p><b>Pencil and paper procedures</b> Children to move onto the expanded long multiplication method when moving onto using more digits; <u>2d x 2d</u> (32 x 56) and <u>4d x 1d/2d</u> (3214 x 8 or 3214 x 39)</p> <p><b><u>23 x 13 = 299</u></b></p> $\begin{array}{r} 23 \\ \times 13 \\ \hline 69 \text{ (3 x 23)} \\ +200 \text{ (10 x 20)} \\ \hline 299 \end{array}$ <p>This leads into Year 6 (Compact long)</p>	<p><b>Pencil and paper procedures</b> Compact long multiplication (formal method): <b><u>23 x 13 = 299</u></b></p> $\begin{array}{r} 23 \\ \times 13 \\ \hline 69 \text{ (3 x 23)} \\ +230 \text{ (10 x 23)} \\ \hline 299 \end{array}$ <p>Compact long multiplication (formal method): <b><u>56 x 27 = 1512</u></b></p> $\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \text{ (7x56)} \text{ (7 x 6 = 42 where the 4 come from)} \\ +1120 \text{ (20x56)} \\ \hline 1512 \\ \hline 1 \end{array}$ <p>When children are confident with long multiplication extend with 3d x 2d, <u>returning to the grid method first, if necessary.</u></p> <p>The prompts (in brackets) can be omitted if children no longer need them.</p>





Division

Year 1

**Sharing**

Requires secure counting skills and so is taught by using lots of practical work.  
 -see counting and understanding number strand  
 Develops importance of one-to-one correspondence  
 See appendix for additional information on x and ÷ and aspects of number

Sharing – 6 sweets are shared between 2 people.  
 How many do they have each?



Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

**Grouping**

This area is also taught using lots of practical work.  
 Sorting objects into 2s / 3s/ 4s etc  
 How many pairs of socks are there?



There are 12 crocus bulbs. Plant 3 in each pot. How many pots are there?  
 Jo has 12 Lego wheels. How many cars can she make?

Year 2

**Grouping**

This area is taught using lots of practical work. Link to counting and understanding number strand  
[Count up to 100 objects by grouping them and counting in tens, fives or twos;...](#)

Find one half, one quarter and three quarters of shapes and sets of objects

6 ÷ 2 can be modelled as:

There are 6 strawberries.

How many people can have 2 each? How many 2s make 6?

12 ÷ 4 can be modelled using practical grouping:

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game.  
 How many teams are there?



**÷ = signs and missing numbers**

$$6 \div 2 = \quad = 6 \div 2$$

$$6 \div \quad = 3 \quad 3 = 6 \div$$

$$\div 2 = 3 \quad 3 = \div 2$$

$$\div \nabla = 3 \quad 3 = \div \nabla$$

Year 3

**÷ = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers.

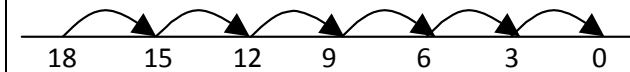
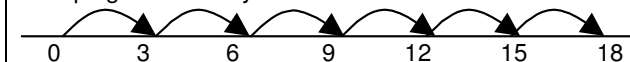
[Understand division as sharing and grouping](#)

18 ÷ 3 can be modelled as:

Sharing – 18 shared between 3 (see Year 1 diagram)

OR

Grouping - How many 3's make 18?



**Remainders**

$$16 \div 3 = 5 \text{ r}1$$

Sharing - 16 shared between 3, how many left over?

Grouping – How many 3's make 16, how many left over?  
 e.g.



Introduce the formal layout using multiplication/division facts that the children

Know (this highlights the importance for them to know their tables:

$$24 \div 3 = 8$$

$$\begin{array}{r} 8 \\ 3 \overline{)24} \end{array}$$

'How many threes are there in twenty four?'



Division		
Year 4	Year 5	Year 6
<p><u>Pencil and paper procedures</u></p> <div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> <math display="block">98 \div 7 = 14</math> <math display="block">\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}</math> </div> <p>Children to continue from year 3, using the formal method introduced without remainders in Year 4.</p>	<p><u>Pencil and paper procedures</u></p> <p><u>Children in year 5 to use the Formal method of short division.</u></p> <p>Children to use this method with;</p> <p>1d ÷ 1d 2d ÷ 1d 3d ÷ 1d 4d ÷ 1d</p> <p style="margin-left: 150px;">} All with remainders</p> <div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> <math display="block">184 \div 8 = 23</math> <math display="block">\begin{array}{r} 23 \\ 8 \overline{) 184} \end{array}</math> <p>...and with remainders:</p> <math display="block">432 \div 5 = 86 \text{ r}2</math> <math display="block">\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \end{array}</math> </div>	<p><u>Pencil and paper procedures</u></p> <p><u>Formal method of short division with remainders</u></p> <p>The remainder can also be expressed as a fraction, 2/5 (the remainder divided by the divisor): <math>435 \div 5 = 86 \frac{2}{5}</math></p> <div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> <math display="block">432 \div 5 = 86 \text{ r}2</math> <math display="block">\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \end{array}</math> </div> <p><u>Formal method of long division</u></p> <div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> <math display="block">496 \div 11 = 45 \text{ r}1</math> <math display="block">\begin{array}{r} 45 \text{ r}1 \\ 11 \overline{) 496} \\ -440 \quad (40 \times 11) \\ \hline 56 \\ -55 \quad (5 \times 11) \\ \hline 1 \quad (\text{remainder}) \end{array}</math> </div> <p>Multiples of the divisor (11) are subtracted from the number (496). Answer is 45 (remainder) 1/11</p> <div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> <math display="block">432 \div 15 = 28 \text{ r}12</math> <math display="block">\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{) 432} \\ -300 \quad (20 \times 15) \\ \hline 132 \\ -120 \quad (8 \times 15) \\ \hline 12 \quad (\text{remainder}) \end{array}</math> </div> <p>Children to use this method with; 1d ÷ 1/2 d 2d ÷ 1/2 d 3d ÷ 1/2 d 4d ÷ 1/2 d</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>The answer is: <math>28 \frac{12}{15}</math> or <math>28 \cdot 8</math></p> </div>