

Numbers Early Learning Goal

- Solve problems, including halving and sharing.

Exceeding judgement

- Solve practical problems that involve sharing into groups of 2, 5 or 10.

Models and Strategies

Children will solve problems in a practical way in the context of real life. They need to see and hear representations of division as **sharing** and **grouping**. Pictorial representations are used alongside concrete apparatus.

Solve real-life problems using the sharing and grouping models of division.

Sharing.

Share real objects (e.g. fruit) equally between a number of children, teddy bears etc. The objects are shared, one per set, until the total is exhausted.

E.g. Eight strawberries are shared equally between 4 children. How many strawberries will each child have?



8 shared between 4 is 2.

Each child will have two strawberries.

Grouping.

Repeatedly subtract equal groups of objects until the total is exhausted.

E.g. I have six socks and I group them into pairs. How many pairs do I have?



I have three groups of two socks.

I have three pairs of socks.

I have 15 apples. If I put 5 apples into each bag, how many bags can I fill?



There are 3 groups of 5 apples.

I can fill 3 bags of apples.

Fractions.

- Find and recognise halves using concrete apparatus (e.g. pizza slices, apple fractions) and corresponding pictorial representations.
- Halve and quarter paper shapes by folding.



Vocabulary of division.

Through a variety of activities and visual representations promote the language of division: *divide, share/share equally, group in pairs, threes..., half/halve, quarter, equals.*

Objectives and additional guidance

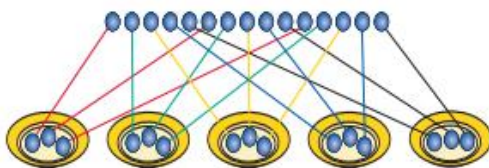
- Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Through grouping and sharing small quantities, begin to understand division and finding simple fractions of objects, numbers and quantities.
- Connect halves and quarters to the equal sharing and grouping of sets of objects and to measures.

Models, Strategies and Methods

Begin to understand division through sharing and grouping small quantities.

Sharing.

Use of concrete apparatus to share **equally**, one at a time. Record pictorially.
 E.g. 15 eggs are shared **equally** between 5 people. Each person will have 3 eggs.



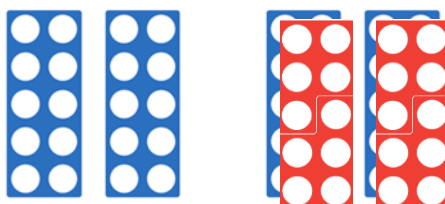
Grouping.

• Use of concrete apparatus and pictorial representations to solve grouping problems.
 E.g. I have 15 apples. If I put 5 apples into each bag, how many bags can I fill?



*There are 3 groups of 5 apples.
 I can fill 3 bags of apples.*

• Use of Numicon and arrays as a representation of grouping.
 E.g. How many groups of 5 are there in 20?



There are 4 groups of 5 in 20.

• Link grouping to times tables / multiplication.



*How many pairs of socks?
 There are 5 groups of 2 in 10.*

Fractions - find, recognise and name halves and quarters of objects, numbers and quantities.

- Find and recognise halves and quarters using concrete apparatus (e.g. pizza slices, apple fractions).
- Halve shapes by folding. Quarter shapes by folding halves.



• Recognise that a half is one of two equal parts, and that a quarter is one of four equal parts.
 Using practical apparatus, find a half by sharing out equally between two; find a quarter by sharing out equally between 4.

• Begin to record and read $\frac{1}{2}$ ("half") and $\frac{1}{4}$ ("quarter"). Link to the clock face (telling the time).
 Extend to three quarters ($\frac{3}{4}$) for some children.

Objectives and additional guidance

- Recall and use division facts for 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs.
- Show that division of one number by another cannot be done in any order.
- Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
- Use a variety of language to describe multiplication and division.
- Begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.
- Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition.
- Begin to relate these to fractions and measures (for example, $40 \div 2 = 20$, 20 is a half of 40). Use commutativity and inverse relations to develop multiplicative reasoning (for example, $4 \times 5 = 20$ and $20 \div 5 = 4$).
- Find $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ ($\frac{1}{2}$) and $\frac{3}{4}$ of a length, shape, set of objects or quantity.

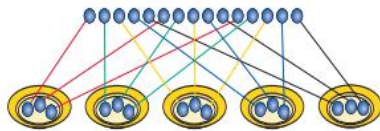
Models, Strategies and Methods

Understand sharing and grouping models of division.

Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations. Understand and use the \div and = signs.

Sharing.

E.g. 15 eggs are shared **equally** between 5 people. Each person will have 3 eggs.



15 shared between 5.
 $15 \div 5 = 3$


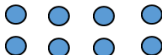
Grouping.

• Children should apply their counting skills to develop some understanding of grouping.

How many 5s in 15? 5  There are 3 groups of 5.
 $15 \div 5 = 3$

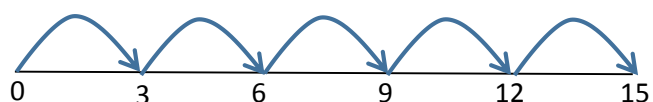
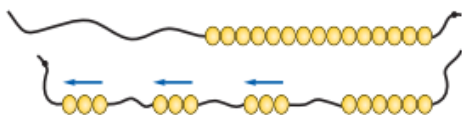
• Continue to use practical apparatus (e.g. Numicon, peg boards, cube towers) and arrays as representations of grouping. E.g. *How many 5s are there in 20?*



There are 4 groups of 3 in 12. 
There are 3 groups of 4 in 12. 

• Use of a bead string / bead bar to count in jumps of the divisor from zero.

E.g. *How many groups of 3 are there in 15?* There are 5 groups of 3. $15 \div 3 = 5$



• Develop an understanding of the inverse operation to link division to times tables / multiplication. Begin to develop fluency in recall of division facts corresponding to the 2, 5 and 10 times tables.

Solve missing number problems. E.g. $2 \times \square = 14$ *How many 2s are in 14?*

Fractions – find one half, one third and one quarter of objects, numbers and quantities.

• Using practical apparatus and images, find a half by sharing out **equally** between two; find one third by sharing out **equally** between three; find one quarter by sharing out **equally** between 4.

Objectives and additional guidance

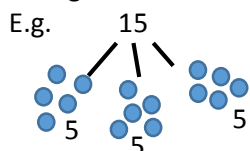
Recall and use division facts for the 3, 4 and 8 multiplication tables.
 Write and calculate mathematical statements for division using the multiplication tables that they know, using mental methods and begin progressing to the formal written method of short division.
 Solve problems, including missing number problems that involve division, positive integer scaling problems (e.g. 4 times lower), and correspondence problems in which m objects are connected to n objects (for example, 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
 Develop efficient mental methods; for example, using multiplication/division facts (e.g. $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).

Models, Strategies and Methods

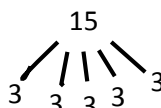
Understand sharing and grouping models of division.

Continue to use grouping and sharing models of division using practical apparatus, arrays and pictorial representations (see Year 2). Extend to divisors appropriate to Year 3.

Recognise that $15 \div 3 = 5$, for example, can be solved both by sharing and grouping.

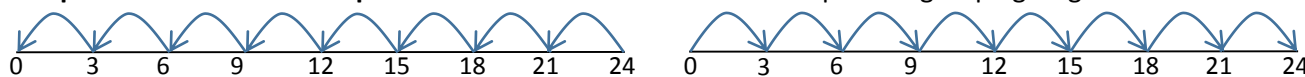


Sharing: 15 sweets are shared equally between 3 children. How many sweets will each child receive?



Grouping: 3 apples are placed in each bag.
 If I have 15 apples, how many full bags of apples will I have?

- **Repeated subtraction or repeated addition** on a number line to represent grouping. E.g. $24 \div 3 = 8$



- Use **arrays** to understand division and its relationship with multiplication. E.g. $18 \div 3 = 6$



becomes

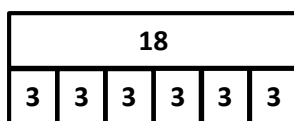


If 3 oranges are placed in each bag, how many full bags will I have? (grouping)

or

If the oranges are shared between 3 bowls, how many will be in each bowl? (sharing)

Represent using the **bar model**:



- Recognise that finding the unit fraction of a quantity is division. E.g. Find $\frac{1}{4}$ of $24 = 24 \div 4$

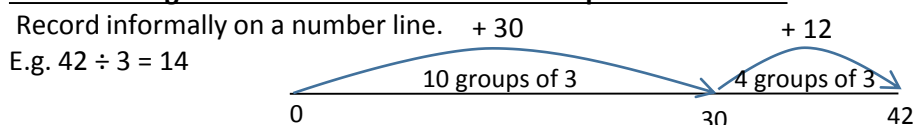
- Introduce **remainders**. E.g. $14 \div 3 = 4$ remainder 2



Mental recall of division facts.

- Continue to develop fluency in recall of division facts corresponding to the 2, 5 and 10 times tables.
 - Develop fluent recall of division facts for 3, 4 and 8 times tables.
- Understand inverse relationships and derive related facts. E.g. $5 \times 3 = 15$; $15 \div 5 = 3$; $3 \times 5 = 15$; $15 \div 3 = 5$ leading to $50 \times 3 = 150$; $150 \div 50 = 3$; $5 = 150 \div 30$ etc.
- Solve missing number problems. E.g. $\square \div 3 = 7$

Use 'chunking' as a more efficient method of repeated addition.



Begin to develop a formal written method.

Some children will use arrays and place value counters to prepare for the formal written method of short division (see Year 4).

Objectives and additional guidance

Recall division facts for all times tables up to 12×12 .

Use place value, known and derived facts to divide mentally, including dividing by 1. Extend to deriving facts for 3-digit numbers (e.g. $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$).

Practise to become fluent in the formal written method of short division with exact answers.

Solve problems involving division, including integer scaling problems (e.g. 4 times lower) and correspondence problems in which m objects are connected to n objects (e.g. 3 cakes shared equally between 10 children).

Find the effect of dividing one- or two-digit numbers by 10 and 100.

Solve two-step problems in context, choosing the appropriate operation, working with increasingly harder numbers.

Solve problems involving increasingly harder fractions to calculate quantities, and divide quantities, including non-unit fractions where the answer is a whole number.

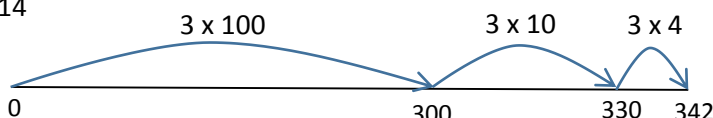
Models, Strategies and Methods

Mental recall of division facts.

- Develop fluency in recall of division facts corresponding to all times tables up to 12×12 and derive related facts. E.g. $30 \div 6 = 5$ leading to $300 \div 6 = 50$; $300 \div 60 = 5$; $50 = 300 \div 6$ etc.
- Understand inverse relationships and solve missing number problems. E.g. $\square \div 8 = 6$; $48 \div \square = 6$
- Recognise that finding the unit fraction of a quantity is division. E.g. Find $\frac{1}{4}$ of $24 = 24 \div 4$

Continue to use 'chunking' as an informal method.

E.g. $342 \div 3 = 114$

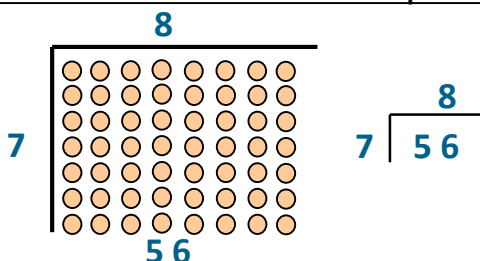


Jottings

$3 \times 100 = 300$
 $3 \times 10 = 30$
 $3 \times 4 = 12$

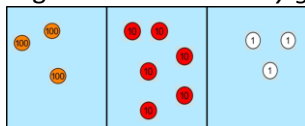
Develop the formal written method of short division to divide numbers up to 3 digits by a one-digit number.

- Use arrays to introduce short division and record as the standard algorithm.

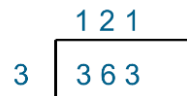


- Model short division using place value counters and record as the standard algorithm.

E.g. $363 \div 3$ How many groups of 3 are there in 363?

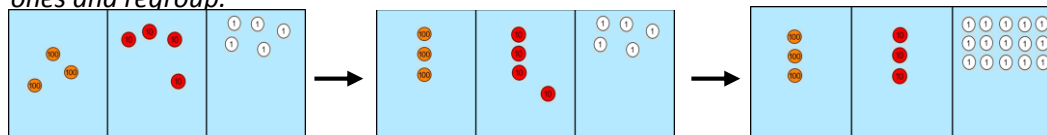


Start with the 100s column: how many groups of 3 can I make? **1**
 Now the 10s column: how many groups of 3 can I make? **2**
 Now the 1s column: how many groups of 3 can I make? **1**



- Introduce remainders. E.g. $364 \div 3 = 121$ remainder 1.

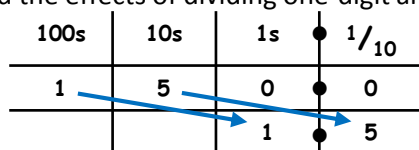
- Progress to **exchanging**. E.g. $345 \div 3 = 115$. 40 is not in the $3 \times$ table so we need to exchange one ten for ten ones and regroup.



Dividing by 10 and 100.

Find the effects of dividing one-digit and two-digit numbers by 10 and 100.

E.g. $150 \div 100 = 1.5$.



Objectives and additional guidance

- Recall and use division facts for all times tables up to 12×12 .
- Identify multiples and factors; find all factor pairs of a number and common factors of 2 numbers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Divide numbers mentally, drawing upon known facts.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Divide whole numbers and those involving decimals by 10, 100 and 1,000.
- Solve problems involving multiplication and division, including: (a) using knowledge of factors and multiples, squares and cubes; (b) scaling by simple fractions and problems involving simple rates.
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
- Use division to convert between units of measure.

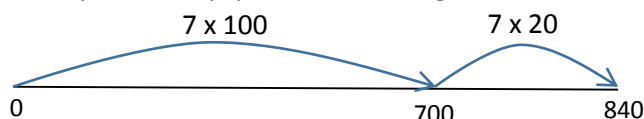
Models, Strategies and Methods

Mental strategies.

- Ensure fluent recall of division facts for all times tables up to 12×12 , and derive associated facts (e.g. $48 \div 8 = 6$; $4.8 \div 8 = 0.6$; $48 \div 0.8 = 60$ etc).
- Solve missing number problems. E.g. $\square \div 8 = 6$; $48 \div \square = 6$
- Identify common factors of given numbers. E.g. Find common factors of 18 and 24.
- Divide whole numbers and decimal numbers by 10, 100 and 1000. Use place value charts to support understanding (e.g. Gattengo chart; *Moving Digits* ITP – see *Multiplication, Year 5*).

Continue to use 'chunking' as a mental calculation strategy.

Record informally on an empty number line. E.g. $840 \div 7 = 120$



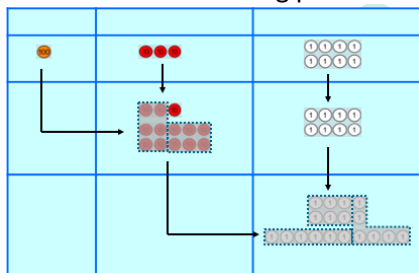
Jottings

$$7 \times 100 = 700$$

$$7 \times 20 = 140$$

Develop fluency in the formal written method of short division to divide numbers up to 4 digits by a one-digit number.

- Reinforce understanding of the standard method using place value counters, as appropriate. E.g. $138 \div 6$



$$6 \overline{) 138} \begin{matrix} 23 \\ \underline{12} \\ 18 \\ \underline{18} \\ 0 \end{matrix}$$

- Develop an understanding of how to express a remainder as a decimal or as a fraction. E.g. $1425 \div 6$

$$6 \overline{) 1425} \begin{matrix} 237 \text{ r}3 \\ \underline{12} \\ 22 \\ \underline{18} \\ 42 \\ \underline{42} \\ 5 \end{matrix}$$

What could I do with the remaining 3? How could I share this between 6 as well?

Express answers as a fraction ($237 \frac{3}{6}$ or $237 \frac{1}{2}$) and as a decimal (237.5).

$$6 \overline{) 1425} \begin{matrix} 237.5 \\ \underline{12} \\ 22 \\ \underline{18} \\ 42 \\ \underline{42} \\ 50 \\ \underline{50} \\ 0 \end{matrix}$$

- Interpret remainders by rounding up or down, as appropriate. E.g. Tickets for a boat trip cost £6 each. How many tickets will I be able to buy with £104 pounds? **17 tickets (rounded down)**
104 scouts are going camping. They are staying in tents that sleep 6 people. How many tents will they need? **18 tents (rounded up)**.

Objectives and additional guidance

- Divide decimal numbers by one-digit whole numbers (initially in practical contexts involving measures and money) using short division, where the answer has up to 2 decimal places.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole numbers, fractions, or by rounding, as appropriate for the context.
- Perform more complex mental calculations with increasingly large numbers, including mixed operations.
- Divide numbers by 10, 100 and 1,000, giving answers up to 3 decimal places.
- Identify common factors, common multiples and prime numbers.
- Use knowledge of the order of operations to carry out calculations involving the 4 operations.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- Divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$].
- Recognise division calculations as the inverse of multiplication.

Models, Strategies and Methods

Mental strategies.

- Ensure fluent recall of division facts up for all times tables up 12×12 , and derive associated facts (e.g. $48 \div 8 = 6$; $4.8 \div 8 = 0.6$; $48 \div 0.8 = 60$).
- Divide whole numbers and decimal numbers by 10, 100 and 1000 (see Year 5).
- Identify common factors of given numbers. E.g. Find common factors of 18 and 24.

Use BIDMAS to solve problems that use knowledge of the order of operations.

E.g. Understand how to solve $8 + 4 \div 2 = 10$ and $(8 + 4) \div 2 = 6$.

Understand division as scaling.

Solve practical scaling problems (e.g. scaling down the ingredients in a recipe).

Formal written methods to divide up to 4 digit numbers by 2 digit whole numbers.

- Develop fluency and deepen understanding of the written method of short division.

$432 \div 5$ becomes $\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$ <p>Answer: 86 remainder 2</p>	$496 \div 11$ becomes $\begin{array}{r} 45 \text{ r}1 \\ 11 \overline{) 496} \\ \underline{44} \\ 56 \\ \underline{55} \\ 1 \end{array}$ <p>Answer: $45 \frac{1}{11}$</p>
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- Develop an understanding of the formal written method of long division. E.g. $432 \div 15 = 28.8$

$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$	Jottings (if needed) - multiples of 15: 15 30 45 60 75 90 105 120
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- Interpret remainders as whole numbers, fractions/decimals, or by rounding, according to the context.
E.g. $1426 \div 8 = 178 \text{ r}2$ or $178 \frac{1}{4}$ or 178.25

Divide proper fractions by whole numbers.

Use diagrams to support understanding. E.g. $\frac{1}{3} \div 2 = \frac{1}{6}$

Also understand that $\div 2$ is the same as $\times \frac{1}{2}$
E.g. $\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

