Bedale CE Primary School

Calculation Strategies Booklet

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Guidance for staff and parents

2014

About this booklet:

This booklet is a guide to the steps that your child will encounter as they progress in maths, from when they start school at four all the way to secondary education. It will cover processes for addition, subtraction, multiplication and division and will focus on mental strategies and written calculation. It is important to note that your child may not learn all of these strategies in their time at primary school.

Calculations:

We want our learners to be happy, successful and confident. The staff are very aware that the maths they, and most likely you, learnt at school looked very different from the maths being taught today. This is mainly due to us being taught methods with little explanation as to where they came from whereas the curriculum today has its foundations built around place value and the understanding of number. We hope then, that by the time children come to use the more compact methods they have an understanding of why they are 'carrying the milk bottle'! We would ask that you support us with your child's learning by modelling with them the method they are currently using in school. If you are ever unsure, please do ask the class teacher for advice.

Another misconception in learning about calculation is that for children to solve a question mentally, all the working must take place in the head. This is not the case, please encourage children to use jottings in order to support their thinking.

Problem solving:

It's not just about being able to solve straight forward calculations, children need to be able to solve real life problems. To achieve our goal as teachers we aim for pupils to be able to apply the processes taught to them in a range of situations. Here are some top tips when helping your child solve problems. You could ask them:

- What process/processes are involved? Is this an add or a take away?
- Can you work it out in your head?
- Do you need to draw a picture or do jottings to help you?
- Would a number line help you?
- Do you need to use a written method?
- Is it appropriate to use a calculator?
- Is the answer sensible? This last one is crucial, it ensures the child is not purely relying on a process but is thinking about whether the answer is appropriate. For example 24 x 4 does not equal 6!

Vocabulary:

The range of vocabulary pupils need to understand and use for each of the processes may surprise you. Obviously pupils in their first year of school will not be familiar with most of the following but by the end of year 6, all of these words are part and parcel of maths lessons:

Addition:	Subtraction:	Multiplication:	Division:
add	take away	lots of	share
altogether	minus	times	shared between
total	less than	groups of	divide
more	fewer than	product	divisible by
more than	subtract	multiply	factor
plus	difference	multiple of	remainder
sum of	decrease	double	halve
increase	How much more is..?	repeated addition	equal groups of

A glossary of key maths vocabulary relating to calculations can be found at the back of this booklet.
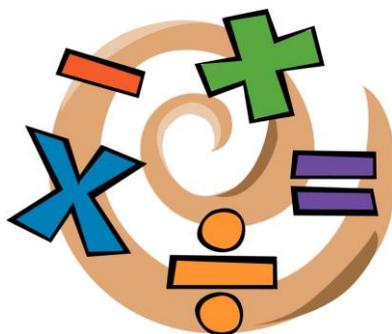
Addition:

The following pages show examples of how we teach addition and how we develop the skills. For the most part, children must be confident at one step before moving to the next. However, previous steps will be reinforced and subsequent steps used to provide challenge, depending on each child's ability.

Learning will be supported by a wide range of resources. These include a 100 square, counters, whiteboards, counting sticks, number lines and a range of interactive place value aids. Please note that stages of progression do not correspond to levels of learning or the year group that your child is in.

Stage 1: (EYFS)

- Counting along a number line

- Singing nursery rhymes

- Counting in real situations with a range of practical equipment

- Using fingers

- Adding objects to a group and using + and = signs.
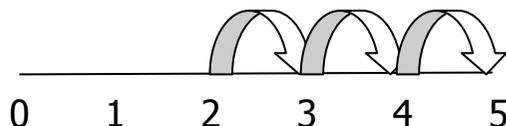
    E.g. ☺☺☺ ☺☺   E.g. At a party I eat 4 biscuits and my friend eats 2.

        3 + 2 = 5   How many biscuits did we eat altogether?

- Using a number line to solve simple problems

    E.g. 2 + 3 = 5

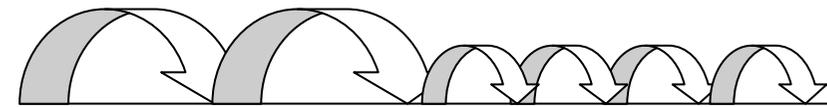```
      0   1   2   3   4   5
```

**Stage 2: (Y1/2)**

- Adding a 2 digit and single digit number by putting the largest number first. Pupils can calculate this mentally, by counting with fingers or by using a number line

- Introduce the concept of a number sentence

- Use ▢ and △ to represent missing numbers

**Stage 3: (Y1/2)**

- Use a number line or 100 square to add two 2 digit numbers. Start with the largest number and partition the 2nd number into tens and units. Use numbers that do not bridge the 10s boundary and then extend to those that do. No answer to exceed 100

E.g. 32 + 24 = 56

32          42          52   53   54   55   56

32          +10         + 10 + 1 + 1 + 1 + 1 = 56

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**Stage 4: (Y2/3)**

- Adding two 2 digit numbers by partitioning both numbers into Tens and Units (ones) and then adding the tens and units separately.

$$35 \ + \ 23$$
$$/ \ \backslash \quad / \ \backslash$$
$$30 \quad 5 \quad 20 \quad 3$$
$$30 + 20 = 50$$
$$5 + 3 = 8$$
$$50 + 8 = \underline{\mathbf{58}}$$

**Stage 4: (Y3/4/5/6)**

- Add numbers using the concise column method. Again, progression of numbers as in stage 4.

E.g.
```
    532   563
  + 334   428
  -----   -----
    866   991

```
⓵

**Extension**: As appropriate for the age and ability of children, and taking into account the method being used, extensions can include placing missing numbers in the question and the use of negative numbers. Also, using the skills within using and applying tasks (using maths knowledge to solve real life problems).

# Subtraction:

The following pages show examples of how we teach subtraction and how we develop the skills. For the most part, children must be confident at one step before moving to the next. However, previous steps will be reinforced and subsequent steps used to provide challenge, depending on each child's ability.

Learning will be supported by a wide range of resources. These include a 100 square, counters, whiteboards, counting sticks, number lines and a range of interactive place value aids. Please note that stages of progression do not correspond to levels of learning of the year group that your child is in.

**Stage 1: (EYFS)**

- Counting backwards out loud

- Singing nursery rhymes and counting songs

- Counting in real situations with a range of practical equipment

- Using fingers

- Taking away objects from a group and counting the number left

- Drawing pictures and crossing out

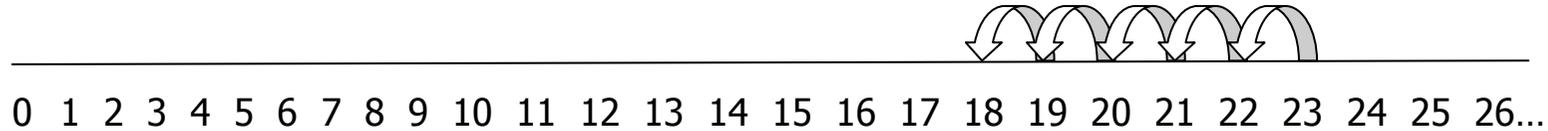    E.g.   $5 - 2 = 3$          ☺ ☺ ☺ ☺ ☺

- Physically finding the difference:

    E.g.   $4 - 3 = 1$       ☺ ☺ ☺ ☺
                              ☺ ☺ ☺

## Stage 2: (EYFS)

Use a number line to count backwards in steps of 1.

E.g.  23 – 5 = 18

0  1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26...

## Stage 3: (Y1/2)

- Start with the largest number and partition the 2$^{nd}$ number into tens and units. Use numbers that do not bridge the 10s boundary and then extend to those that do. No answer to exceed 100

E.g.  44 – 21 = 23

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**Stage 4: (Y1/2/3)**

Develop the idea of finding the difference by starting with the smallest number and counting on to the largest number. Progression of this number line method as follows:

- o TU-U

- o TU- TU

- o HTU-HTU

- o ThHTU - ThHTU

- o Mixture of the above

Children are encouraged to jump in separate steps of hundreds, tens & units to ensure that the final addition is easier and completed with accuracy.

E.g.    $164 - 89 = 75$
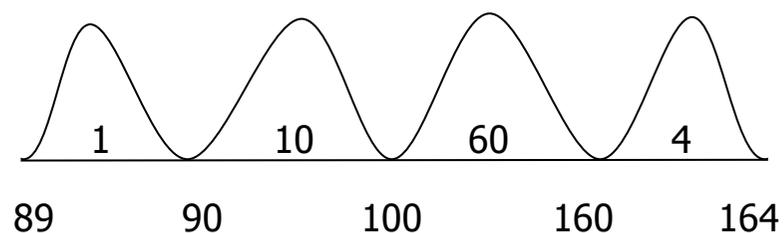


$60 + 10 + 4 + 1 = 75$
(children use appropriate jottings / methods according to their ability)

## Stage 5: (Y3/4)

Begin to use formal column method, initially with no decomposition. Begin by explaining in terms of place value. Follow stage 4 for progression of numbers.

E.g- 453-212= 241

```
  453
 -212
 ____
  241
```

Children are also encouraged to check answers using the inverse operation.

Stage 6: (Y4/5/6)

Move to the column method that requires decomposition. Again, ensure understanding of place value. Follow stage 4 for progression of numbers. Ensure pupils understand how zero is used as a place holder and that in some circumstances a number line may be a more appropriate method to use.

```
  5
  6₁4 3
  4 8 3
  _____
  1 6 0
```

Extension: As appropriate for age and ability of children, and taking into account the method being used, extensions can include placing missing numbers in the question and answer and the use of negative numbers. Also, using the skills within using and applying tasks.

Multiplication:

The following pages show examples of how we teach multiplication and how we develop the skills. For the most part, children must be confident at one step before moving to the next. However, previous steps will be reinforced and subsequent steps used to provide challenge, depending on each child's ability.

Learning will be supported by a wide range of resources. These include a 100 square, picture arrays, multiplication squares, counters, whiteboards, counting sticks, number lines and a range of interactive aids. Please note that stages of progression do not correspond to levels of learning or the year group your child is in.

Stage 1: (EYFS/Y1)

Introduce counting in 2s, 10s and then 5s. Use this in a range of contexts. Count backwards as well as forwards.

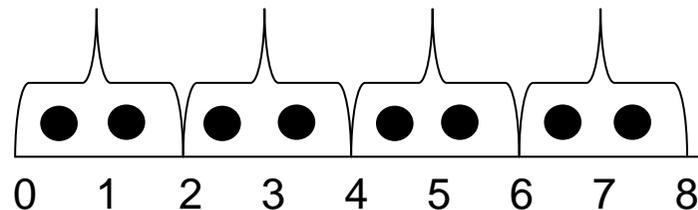Teach doubling of numbers up to 10.

Stage 2: (Y1/2)

Introduce repeated addition.   This can also be demonstrated on a number line.

E.g.   Each child has 2 feet.

How many feet to 4 children have?

Stage 3: (Y1/2)

Introduce arrays to model multiplication and the use of the x sign in number sentences.

E.g.   5 x 3 = 15

☺☺☺☺☺   ☺☺☺   Use the image to help pupils understand that
☺☺☺☺☺   ☺☺☺   5 x 3 is the same as 3 x 5.
☺☺☺☺☺   ☺☺☺
☺☺☺
☺☺☺

Use arrays to help solve real life problems, using lots of practical resources for support.

Stage 4: (Y2/3)

Show children that the inverse of multiplication is division. Use arrays to demonstrate visually.

Using this knowledge, use symbols to find missing numbers.

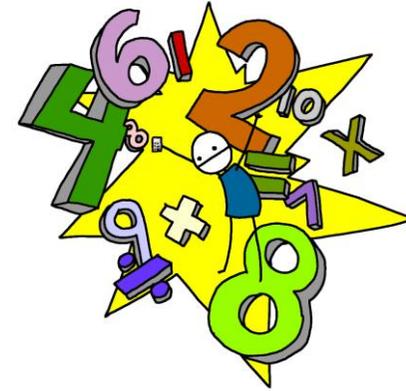E.g.   10 x △ = 70   ▢ ÷ 4 = 2

Stage 5: (Y3-6)

Use the grid method to multiply, focusing on place value when multiplying by 10. Progress through the stages of difficulty as follows:

- o TU x TU

- o HTU x TU

E.g.   56 x 32

	50	**6**	
30	1500	180	= 1680
2	100	12	= 112
			1792

Addition strategies used in line with children's

understanding to date.

Stage 6: (Y3-6)

Introduce the column method for addition. Initially, model alongside the grid method and ensure that no answer is greater than 10.

Multiplying by 1 digit number (Y4/5)   Multiplying by 2 digit number (Y5/6)

E.g. Initial stage: 32 x 13   = 416

$$32$$

	30	**2**		
10	300	20	=	320
3	90	6	=	96

$$13$$

320
96
416

416
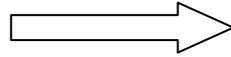
These stages can be done in either order as long as children spot the link with the grid method.

Develop to: 436 x 23

```
   436
    23
  1308    (x3)
  1 1
  8720    (x20)
    1
 10028
    1
```


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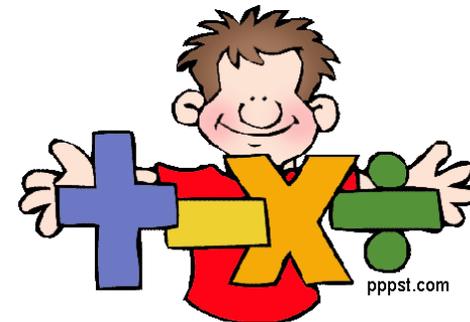
Division:

The following pages show examples of how we teach division and how we develop the skills. For the most part, children must be confident at one step before moving to the next. However, previous steps will be reinforced and subsequent steps used to provide challenge, depending on each child's ability.

Learning will be supported by a wide range of resources. These include picture arrays, multiplication squares, counters, whiteboards, counting sticks, number lines and a range of interactive aids. Please note that stages of progression do not correspond to levels of learning.

While fractions of numbers are a key part of division, the detail of progression of fractions is not included in this booklet. As a quick guide, the initial finding of halves and quarters of numbers would begin at around stage 5.

The rules of divisibility are not included either. These can be taught in line with the tables from Y1/2 upwards.

Stage 1: (EYFS)

Keep it practical. Lots of physical sharing into equal groups or sets.

Stage 2: (EYFS/Y1)

Mental knowledge of halving numbers up to 20.

Using real objects to solve simple problems.

Stage 3: (Y1/2)

Finding how many lots of 2, 5 or 10 are in a given number. Link to arrays & multiplication. Begin to introduce the ÷ symbol.
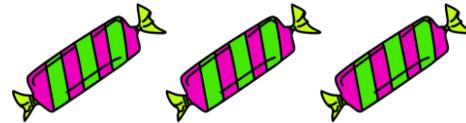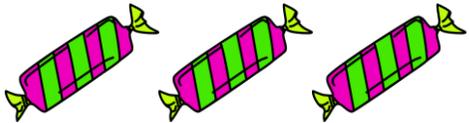
Stage 4: (Y1/2)

Use pictures and objects to solve problems that involve techniques of grouping and sharing.
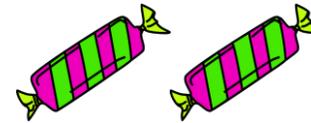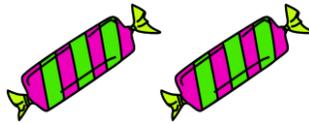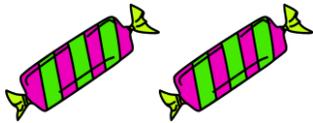
E.g.   6 sweets are shared between 2 children. How many sweets do they each get?

Sharing between 2

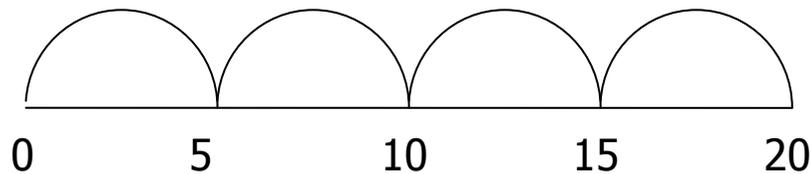E.g. There are 6 sweets. How many children can have 2 each?

Grouping into 2s

Stage 5: (Y2/3)

Teach pupils the written method using the number line. Start with simple steps of one jump at a time. Link to repeated subtraction.

E.g. $20 \div 5 = 4$

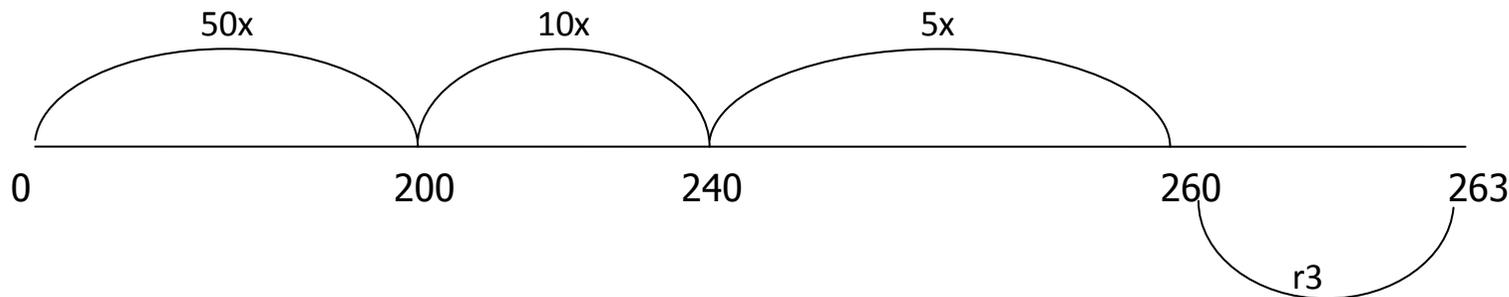Explain that we are looking to see how many lots of 5 fit into 20.

0   5   10   15   20

Stage 6: (Y3/4/5)

Develop this method making more efficient jumps using a fact box

E.g. $263 \div 4 = 65$ rem 3

50x   10x   5x

0   200   240   260   263

r3

Fact box:

$1 \times 4 = 4$
$10 \times 4 = 40$
$2 \times 4 = 8$
$20 \times 4 = 80$
$5 \times 4 = 20$
$50 \times 4 = 200$

Stage 7: (Y5/6)

Only when children are completely clear with all other compact methods and place value should they be taught the compact method for division. Follow the steps of progression as in stage 5. Instead of the fact box for the number line method, pupils may find it helpful to jot down the 1st multiples of the divisor.

E.g. $576 \div 16 = 36$

$$
\begin{array}{r}
3\ 6 \\
\hline
16\ )\ 5\ 7_96
\end{array}
$$

1 x 16 = 16
2 x 16 = 32
3 x 16 = 48
4 x 16 = 64
5 x 16 = 80

Extension: To extend division, divide whole numbers into decimal numbers, explore recurring numbers as answers, convert answers with remainders into fractions and decimals, include missing numbers in questions, use the inverse to check answers with remainders and apply knowledge to a range of situations.

Although understanding calculations and their place value are fundamental to maths, there are other areas that are key to gaining an overall understanding of the subject. Here are some ideas to support primary age children outside of school.

Real Life Problems:

- Go shopping with your child to buy two or three items. Ask them to work out the total amount spent and the amount of change you will get.

- Buy items with a percentage extra free. Calculate how much weight/liquid you are getting free.

- Plan an outing. Use a time line to work out what time you need to set off, how much money you need to take…

- Use a TV guide to work out the length of programmes. Calculate how long you spend watching TV in a day/week.

- Plan a bus/train journey using a timetable. Look at how long the journey should take, how many changes need to be made. If you go on the journey, discuss whether the timings were right!

- Work together to scale a recipe up or down to feed the right amount of people.

- Plan a meal or a party with a given budget.

These are just a few ideas to give you a start. Involving children in problem solving activities helps to motivate them in their learning.

Shape and Measure:

- Choose a shape for the week (e.g. cuboid). Describe the shape and see if you can spot it in the environment.

- Play 'guess my shape'. Think of a shape and your child has to guess it using yes/no questions only.

- Go on a right angle hunt around your home. Can you spot angles that are bigger/smaller.

- Look for symmetrical objects, complete symmetrical pictures/patterns.

- Make a junk model, discuss the different shapes, their sizes and properties.

- Practise measuring the lengths/heights of objects, estimating beforehand. Younger children could measure in cubes, hands before using rulers & tape measures in cm and m.

- Cook at home, weighing and measuring liquids. Talk about what the divisions on the scales mean.

- Choose some items from the cupboard. Try to put them in order of weight then check by looking on the packets or weighing them.

- Practise telling the time, use both digital and analogue clocks.

- Estimate 1st then use a stop watch to time challenges.

Early Counting ideas:

- Practise counting the number names. When children are confident start from different numbers. E.g. 4, 5, 6… Count forwards and backwards.

- Practise writing these numbers in a range of media: paint, shaving foam, sand…

- Sing familiar number rhymes together, you can buy puppets, CDs… to support you with this.

- Give children a range of items to touch and count. Encourage then to touch each one as they count.

- Count things you can't touch or see. E.g. claps, number of oranges in a bag.

- Play any games that involve counting. Snakes and ladders being one of the classics.

- Look for numbers in the environment.

- Cut out numbers from magazines, papers, cards and ask children to put them in order.

- Make mistakes when chanting, counting, ordering… Can your child spot the deliberate mistake.

- Choose a number of the week. See how many times you can find it that week. Find that number of pens, toys.

Practising Number Facts:

- Look at the bookmark in your child's reading folder to find the fact that your child is learning. Practise this for a few minutes a day using a range of vocabulary.

- Have a 'fact of the day/week' and pin it up around the house for children to find and recite in a range of voices (quiet, loud, monster..) Can they recall it at the end of the time?

- Play ping pong to recall number bonds. Aim for instant response without the use of fingers. This can be used for bonds to 10, 20, 100…

- Throw 2 dice. Can children find the total, difference, product?

- Using playing cards, turn two cards at a time. Your child has to add, subtract, multiply (what ever you decide) the numbers to keep them. How many can they collect in 2 mins?

- Given the answer, how many questions can children think of in a given time?

- Give your child a number fact and ask them what else they know. E.g. given $6 + 5 = 11$ they know that:
$5+6 = 11, 11-5 = 6, 11-6 = 5, 16+5 = 21, 50+60 = 110…$

Glossary:

Arrays: A visual representation to assist the understanding of multiplication and division.

Blank number line: A horizontal line where the children can insert their own numbers to solve calculations.

Bridging or crossing the tens boundary: Adding or subtracting across a multiple of 10, or counting on in multiples of 10.

Complement: Another term for a number bond.

Compact method: The traditional method of solving a calculation.

Decomposition: A strategy used in subtraction, of moving 100s or 10s to facilitate the calculation.

Difference: The difference between two numbers is the amount between them.

Digits: A single symbol that represents a counting number. 0-9 are one digit numbers.

Expanded method: A written method that shows the intermediate stages in the calculation.

Factor: A whole number that divides exactly into another number.

Grid method: Splitting numbers into hundreds, tens and units for multiplication.

Grouping: Dividing things into equal groups.

Inverse operations: The notion that each operation has an opposite. Addition/subtraction, multiplication/division.

Jotting: Any method of recording numbers/calculations that is not formalised.

Kay facts (number facts): Easily remembered pieces of information used to help with more complicated calculations.

Lattice method: Development of the grid method that allows a more efficient multiplication with decimals.

Mental calculations: Calculations done in the head, sometimes with minimal jotting.

Most/Least significant digit: In place value the most significant digit is placed further to the left, having the highest value. The least significant digit is the one placed furthest to the right, having the lowest value.

Multiple: 5, 10, 15, 20... are all multiples of the 5x table.

Negative number: A number that is below zero, often used in the context of temperature.

Number bonds: All the pairs of numbers that total to a given number. E.g. number bonds to 10 are 7+3, 2+8, 10+0 etc.

Number line: Horizontal line with ordered numbers, used to support calculation.

Number sentences: Horizontal calculations using digits and symbols, e.g. 5 + 3 = 8

Number stories: Putting calculations in the context of a story. E.g. 3 + 4 = 7. Jim has three rabbits and Jane has 4 guinea pigs, how many animals do they have altogether?

Operations: The four types of calculation: + - x ÷

Partitioning: Splitting numbers into 100s, 10s and units to help with calculation.

Pattern: A recurring sequence.

Place holder: The use of the numeral 0 in an empty column to ensure that the other digits are in the correct column and hold their value. E.g. 407, 530.

Place value: The value of a digit depending on its place in a number.

Product: The answer when two numbers are multiplied.

Recombining: Bringing numbers back together after partitioning. E.g. 30 + 5 = 35.

Remainder: The amount left over after dividing a number.

Repeated addition: A simple introduction to multiplication.

Repeated subtraction: A simple introduction to division.

Sharing: Dividing things into equal groups by distributing one at a time.

Vertical/column method: Any written method set out in a vertical format.

I $\heartsuit^2$ Maths