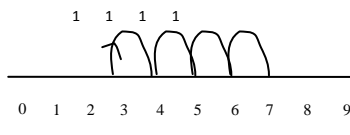
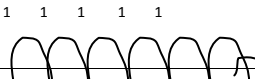


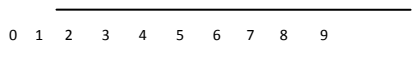
PROGRESSION IN ADDITION AND SUBTRACTION


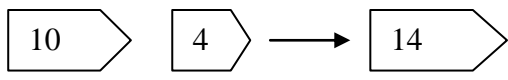

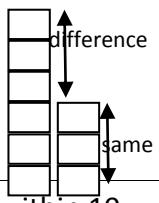
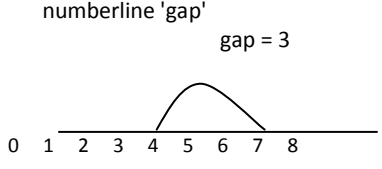
This progression runs alongside general understanding of numbers and their relationships.

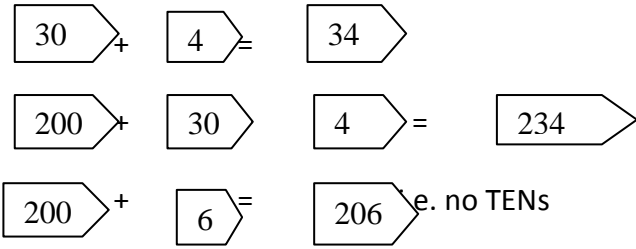
As of September 2014 column headings will be read as HT1 alongside HTU

N.B -Numicon and its resources introduced January 2015 to support and enrich already established maths curriculum.

	Progression *introduced as a result of Curriculum 2014	Vocabulary											
0a	<p>Vocabulary and language of adding and taking away</p> <p>TRACK with all numbers showing</p> <table border="1" style="margin-left: 40px;"> <tr> <td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td> </tr> </table> <p>E.g. Stand on 4. What number are you standing on if..... You take one more step? You add one more step? You go back one step? You take away one step? How many steps would get you back to zero? How many steps would get you to 10? What numbers are your next door neighbours?</p>	0	1	2	3	4	5	6	7	8	9	10	<p>number names inc. zero more, less / fewer count, count on, count in ones, count back how many too many, too few, not enough first, second, third... biggest, smallest, fewest... before, after, next add, sum, altogether, one more, how many more, equals, is the same as 'one more is the next counting number'. take away, left over subtract SYMBOLS: 1 to 10, +, -, =</p>
0	1	2	3	4	5	6	7	8	9	10			
0b	<p>TRACK with some numbers hidden with example questions as above.</p> <table border="1" style="margin-left: 40px;"> <tr> <td>0</td><td>1</td><td></td><td></td><td>4</td><td></td><td>7</td><td></td><td></td><td>10</td> </tr> </table>	0	1			4		7			10		
0	1			4		7			10				
0c	<p>Compare two sets/groups Which has more? Which has less/ fewer?</p>												
0d	<p>Add 1 / 2 more – practically (cubes on a plate, coins) - on the track Go back 1 / 2; take away 2 – practically (cubes on a plate, coins) - on the track</p>												
1a	<p>Move from track to numberline: Introduce SYMBOLS + = - (later) NOTE: on a number line (as opposed to a track), you are on a SPOT not a 'patch'.</p> <p>ADD (practically) as 'combining 2 or more groups' AND as 'adding to the first number' (aiming for counting on). SUBTRACT (practically) as 'take away' initially. Same questions as for track in 0a</p>	<p>further number names count in tens, up and back move forward and backwards</p>											
1b	<p>counting on, on a numberline $3 + 4$</p> 												
1c	<p>'hopping back' on a numberline $7 - 6$</p> 												



1ci	<p>'Grab' numbers to show structure for TEEN numbers DIENES' BASE 10 for 'teen' numbers</p> <p>14 → </p> <p>Place value cards </p> <p>KNOW bonds $14 = 10 + 4$, $10 + 4 = 14$</p>	TENS, UNITS ONES digit 'teens' plus, total sign
1cii	<p>READ NUMBER SENTENCES + and - - relate to practical activities e.g. $3 + 4 = 7$ and $7 = 3 + 4$ - see a number sentence and create a story for it</p>	number sentence addition, subtraction
1d	<p>Put larger number first when adding $3 + 4 = 4 + 3$ 'you're still on the same number in the end'.</p> 	
<p>ABOUT NOW PUPILS MOVE FROM PLAIN PAPER TO SQUARED PAPER FOR FORMAL RECORDING OF WORK</p>		
1e	<p>SUBTRACTION as comparing two numbers to find the DIFFERENCE (i.e. The GAP between them) with towers of cubes</p>  <p>numberline 'gap'</p>  <p>$6 - 3$</p>	<p>difference (gap between)</p> <p>[- can be read as subtract, minus or take away]</p>
1f	<p>Add 3 numbers within 10</p> <ul style="list-style-type: none"> - by combining sets and - by counting on 	
1g	<p>WORD PROBLEMS</p> <ul style="list-style-type: none"> - acting out addition word problems - acting out subtraction word problems 	act it out model it with cubes

1h	Use □ or Δ for MISSING NUMBERS (+ and –) to 10 and then to 20 <ul style="list-style-type: none"> - use known bonds, - equaliser - number track (optional) - numberline 	What number is missing?
2a	Partitioning and recombining numbers Place Value 'arrow cards' 	further number names calculate 1,2,3 digit number partition, recombine rule for a pattern arrow/place value cards RULE: when adding it's quicker to start from the larger number RULE: a number sentence must balance / must be the same amount on both sides RULE: you can swap numbers around when adding and get the same answer BUT <u>NOT</u> for subtraction;
	Add 3 numbers within 20 <ul style="list-style-type: none"> - by combining sets and - by counting on 	RULE: you can subtract by taking away or finding the difference
	ADD 3 NUMBERS by pairing bonds, as 'adding can be done in any order' e.g. $3 + 4 + 7 = 10 + 4$	RULE: you can subtract by taking away or finding the difference
	EXPLORE EQUATIONS WHICH BALANCE – initially with 2 small numbers on each side, then more than 3 numbers if child is ready $2 + 5 = 4 + 3$ using equaliser, then mental arithmetic	RULE: you can add numbers in any order BUT <u>NOT</u> for subtraction
	Further work on SUBTRACTION AS DIFFERENCE (by comparing)	operation addition minus, sign, subtraction; place value, partition, combine round, nearest
	Use □ or Δ for MISSING NUMBERS (+ and –) to 30	H, T, U/1

2b **TU ± U (T1 ± 1)** e.g. 42 + 6, 46 - 4

Dienes'

Spike Abacus

2c On numbered numberline

42 + 6

46 - 4

symbols < and >

2d **TU + U (T1 + 1)** e.g. 46 + 7

2e counting on 4653

2f numberline as above;

partitioning $40 + 6 + 7 = 40 + 13 = 53$ OR

redistribution $43 + 10$ (moving 3 over) OR $50 + 3$ (moving 4 over)

'bridging' through TEN on a numberline

i.e. Only 2 hops allowed. MUST land on next whole TEN.
So partition the UNITS value using knowledge of bonds within 10.

(adding) I can exchange 10 of these for 1 of those next door

2g **TU - U (T1 + 1)** (over the ten, decomposition) e.g. 42 - 9

Counting back - on a numberline

Dienes'

'bridging' e.g. 42 - 9 → 42 - 2 - 7

(subtracting) 1 exchange one TEN for 10 UNITS/ONES

See a NUMBER SENTENCE and create a story for it

- act out / model / sketch addition and subtraction WORD PROBLEMS
- start LIST OF WORDS which might mean you have to add (more, add, plus, altogether....) or subtract (minus, less than.....)

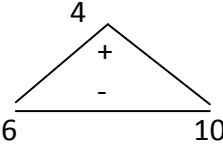
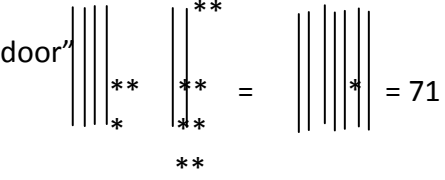
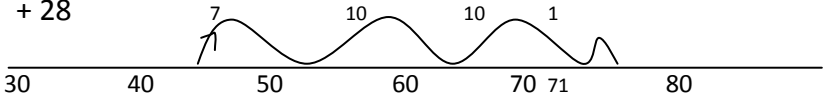
NOTE the words that are on BOTH lists (e.g. more)

INVESTIGATE PATTERNS

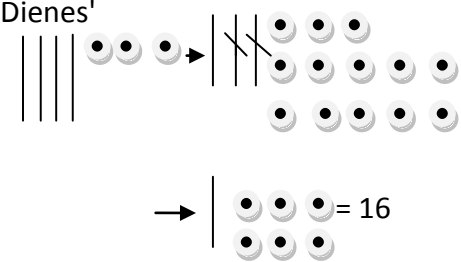
e.g. Patterns: list systematically all bonds to 10 [0+10, 1 + 9,..... 10+0]


Patterns: 3 + 4 + 7, so 30 + 40 = 70

Patterns: 3 + 4 = 7 so 13 + 4 = 17

2h	<p>Create number sentences</p>  $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$																															
	<p>ADDITION AND SUBTRACTION ARE INVERSE OPERATIONS - "you can get back to where you started from"</p> <p>- function machines In <u>+2</u> <u>2</u> Out</p> <p> 3 → 5 → 3</p> <p> ? → ? → 6</p> <p>- calculating $3 + 5 = 8$ then $8 - 5 = 3$</p>	Inverse operation																														
2h*	<p>Column recording</p> <table style="margin-left: 100px;"> <tr> <td></td> <td>T1</td> <td>T1</td> <td>T1</td> </tr> <tr> <td></td> <td>6</td> <td>13</td> <td>37</td> </tr> <tr> <td>+ and -</td> <td><u>+4</u></td> <td><u>+ 4</u></td> <td>- <u>14</u></td> </tr> <tr> <td></td> <td><u>10</u></td> <td><u>27</u></td> <td><u>23</u></td> </tr> </table>		T1	T1	T1		6	13	37	+ and -	<u>+4</u>	<u>+ 4</u>	- <u>14</u>		<u>10</u>	<u>27</u>	<u>23</u>	Columns rows														
	T1	T1	T1																													
	6	13	37																													
+ and -	<u>+4</u>	<u>+ 4</u>	- <u>14</u>																													
	<u>10</u>	<u>27</u>	<u>23</u>																													
<p>2i</p> <p>2j</p> <p>2k</p> <p>2k*</p>	<p>TU + TU (T1 + T1)</p> <p>43 + 28 with Dienes</p> <p>door"  = 71</p> <p>"Exchange 10 units/ones for one TEN next</p> <p>OR</p> <table style="margin-left: 100px;"> <tr> <td>43</td> <td>+</td> <td>28</td> <td></td> </tr> <tr> <td>40 + 3</td> <td>+</td> <td>20 + 8</td> <td></td> </tr> <tr> <td>60</td> <td>+</td> <td>11</td> <td>= 71</td> </tr> </table> <table style="margin-left: 100px;"> <tr> <td>43</td> <td>+</td> <td>20</td> <td>+</td> <td>8</td> <td></td> </tr> <tr> <td>63</td> <td>+</td> <td>8</td> <td>=</td> <td>71</td> <td></td> </tr> </table> <p>43 + 28</p>  <p>ANY confident jump sizes, hopefully bridging if necessary</p> <p>column recording - expanded</p> <table style="margin-left: 100px;"> <tr> <td></td> <td>43</td> </tr> <tr> <td>+</td> <td><u>28</u></td> </tr> <tr> <td></td> <td>11</td> </tr> </table>	43	+	28		40 + 3	+	20 + 8		60	+	11	= 71	43	+	20	+	8		63	+	8	=	71			43	+	<u>28</u>		11	
43	+	28																														
40 + 3	+	20 + 8																														
60	+	11	= 71																													
43	+	20	+	8																												
63	+	8	=	71																												
	43																															
+	<u>28</u>																															
	11																															

TU - TU (T1 - T1) with decomposition e.g. 43 - 27

2l Dienes'  "can't take away the 7 so first exchange one TEN for 10 of those next door, then take away the 7"

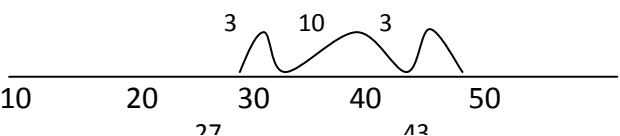
→  = 16

NB. With apparatus can take away the 20 OR the 7 first with no problems.

2m PV partitioning of 2nd number
 $43 - 27 = 43 - 20 - 7 = 23 - 7 = 16$

2n Find difference on a numberline

$43 - 27$ $3+10+3=16$



3a **PARTITION AND RECOMBINE TH H T U/1**
 possibly with arrow cards
 $4321 = 4000 + 300 + 20 + 1$

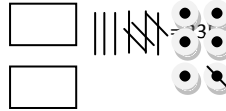
3, 4 digit number
TH, H, T, U/1

HTU ± TU (HT1 ± T1) e.g. 266 ± 31

3b Dienes' 266 + 31



266 - 31



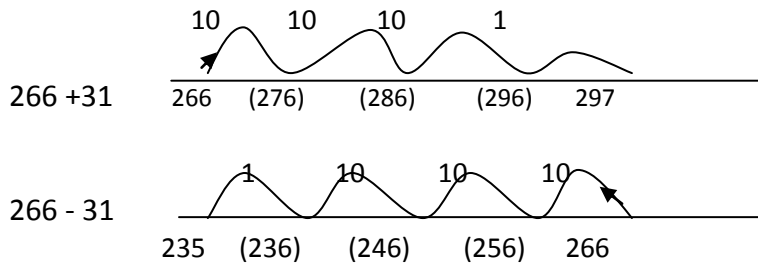
3c PV partitioning

$$\begin{array}{r}
 266 + 31 \\
 266 + 30 + 1 \\
 \hline
 296 + 1 + 279
 \end{array}
 \qquad
 \begin{array}{r}
 266 - 31 \\
 266 - 30 - 1 \\
 236 - 1 = 235
 \end{array}$$

3d Redistribution

$$\begin{aligned}
 266 + 31 &= 267 + 30 \text{ (moving one over)} \\
 &= 297
 \end{aligned}$$

3e BLANK numberline



3f Expanded vertical columns

$$\begin{array}{r}
 266 \\
 + \underline{31} \\
 7 \\
 90 \\
 + \underline{200} \\
 \underline{297}
 \end{array}$$

NOTE: can start with
H or U/1 column at this stage.
BUT may start saying it's
more efficient later to start
in the right hand column.

$$\begin{array}{r}
 266 \\
 + \underline{31} \\
 200 \\
 90 \\
 + \underline{7} \\
 \underline{297}
 \end{array}$$

subtraction

$$\begin{array}{r}
 266 \\
 - \underline{31} \\
 5 \\
 30 \\
 + \underline{200} \\
 \underline{235}
 \end{array}$$

RULE: a number sentence must balance / must be the same amount on both sides;

multiple plus, increase, decrease

carry ONE ten column addition,

subtraction (recording uses squared paper)

RULE: you must never have 2 numbers in one column (square)

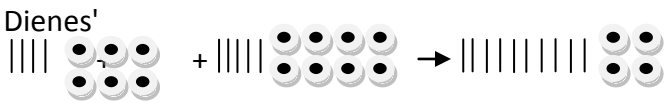

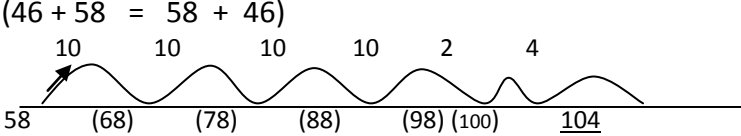



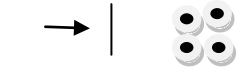
Add several 2 digit numbers


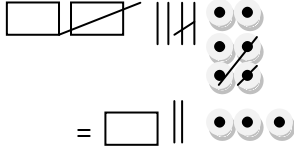
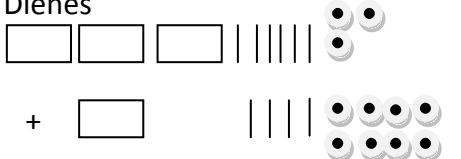

FURTHER PATTERNS

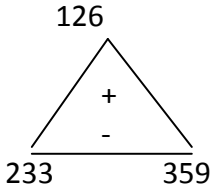
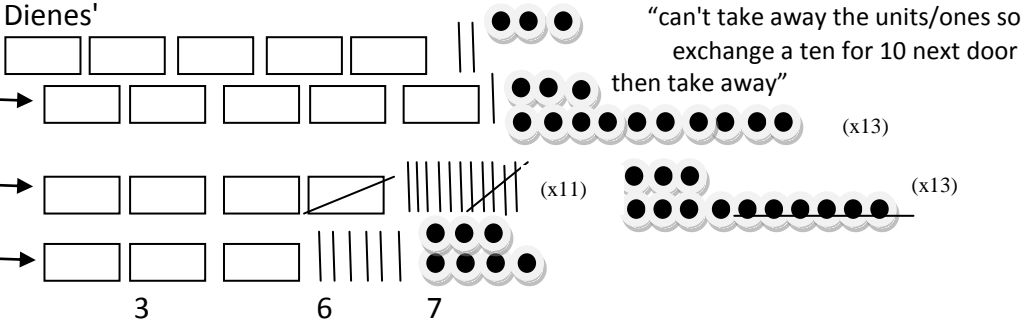
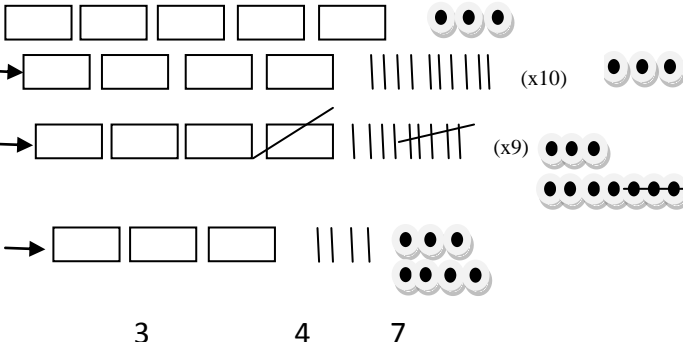
e.g. 30 + 40 + 70, so 300 + 400 = 700

$$35 + 42 = 77 \text{ so } 45 + 42 = 87$$

Patterns you know one fact, so you know several more

	<p>MISSING NUMBERS use □ or Δ for missing numbers to 50 - include reversing equation e.g. $35 = 17 + \Delta$, $17 + \Delta = 35$</p> <p>use □ or Δ for missing multiples of 100 - include reversing equation e.g. $100 + \Delta = 300$ can be rewritten $300 = 100 + \Delta$</p>	<p>Remember rule: you can add in any order but you cannot change subtraction.</p>
	<p>WORD PROBLEMS THROUGHOUT See a number sentence and create a story for it - acting out / model / sketch addition and subtraction word problems</p> <p>WORD LIST - words which might mean you have to add (more, add, altogether, total) or subtract (minus, difference, less than, more). NOTE: some words (e.g. more) might be in BOTH lists.</p>	
<p>3g</p>	<p>TU + TU (T1 ± T1) e.g. 46 + 58</p> <p>Dienes' </p> <p>→  = 104</p> <p>“Exchange 10 units/ones for 1 next door Exchange 10 tens for 1 next door”</p> <p>PV Partitioning</p> $\begin{array}{r} 46 + 58 \\ 46 + 50 + 8 \\ \hline 96 + 8 = 104 \end{array}$ <p>Blank numberline (46 + 58 = 58 + 46)</p>  <p>Column addition (expanded)</p> $\begin{array}{r} 46 \\ + 58 \\ \hline 14 \\ \hline 100 \\ \hline 104 \end{array}$	
<p>3h</p>	<p>TU - TU (T1 - T1) with decomposition, e.g. 52 - 38</p> <p>Dienes' </p> <p>→ </p> <p>so</p> <p>door </p> <p>→ </p> <p>= 1 4</p> <p>“can't take away the units/ones exchange a ten for 10 next then take away”</p>	

3i	<p>HTU ± HTU (HT1 ± HT1) e.g. 246 ± 123</p> <p>Dienes' 246 + 123  = 369</p> <p>PV partitioning 246 + 123 246 + 100 + 20 + 3 = 346 + 20 + 3 = 366 + 3 = 369</p> <p>Expanded column 246 $\begin{array}{r} 246 \\ + 123 \\ \hline 9 \\ 60 \\ + 300 \\ \hline 369 \end{array}$</p> <p>Dienes' 246 - 123  = $\begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array}$ 1 2 3</p> <p>expanded column 246 (optional) $\begin{array}{r} 246 \\ - 123 \\ \hline 3 \\ 20 \\ + 100 \\ \hline 123 \end{array}$</p> <p>standard column 246 $\begin{array}{r} 246 \\ - 123 \\ \hline 123 \end{array}$</p>	
	<p>MISSING NUMBERS use □ or Δ for missing numbers to 100 - include reversing equation</p> <p>e.g. 95 = 37 + Δ, 37 + Δ = 95</p> <p>use □ or Δ for missing multiples of 1000, if ready - include reversing equation</p> <p>e.g. 3000 = 1000 + Δ, 1000 + Δ = 3000</p>	TH, H, T, U/1 million numeral
4	<p>Compacted standard column decomposition TU - TU (T1 - T1)</p> $\begin{array}{r} 52 \\ - 38 \\ \hline 14 \end{array}$	
4a	<p>HTU + HTU (HT1 - HT1) e.g. 363 + 148</p> <p>Dienes'  +  = $\begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \end{array}$ = 5 1 1</p> <p>Expanded column addition 363 $\begin{array}{r} 363 \\ + 148 \\ \hline 11 \\ 100 \\ + 400 \\ \hline 511 \end{array}$</p>	

4b	<p>4 linked equations</p>  $126 + 233 = 359$ $233 + 126 = 359$ $359 - 126 = 233$ $359 - 233 = 126$	<p>RULE: a number sentence must balance / it can be called an equation</p>
4b*	<p>Standard column addition</p> $\begin{array}{r} 363 \\ +148 \\ \hline 511 \\ 1\ 1 \end{array}$	<p>(adding) I can exchange 10 of these for one of those <u>next door</u></p>
4c	<p>HTU - HTU (HT1 - HT1) with exchange e.g. 523 - 156 i.e. CAN take from 'next door'</p> <p>Dienes'</p>  <p>“can't take away the units/ones so exchange a ten for 10 next door then take away”</p> <p>(x13)</p> <p>(x11)</p> <p>(x13)</p> <p>3 6 7</p> <p>(N.B. can be done in any order)</p> <p>PV expanded partitioning (optional)</p> $\begin{array}{r} 500 + 20 + 3 \rightarrow 500 + 10 + 13 \rightarrow 400 + 110 + 13 \\ - 100 + 50 + 6 \quad - 100 + 50 + 6 \quad - 100 + 50 + 6 \\ \hline 300 + 60 + 7 = \\ \underline{\quad\quad\quad} \\ 367 \end{array}$	<p>(subtracting) I can exchange 1 of these for 10 of those <u>next door</u></p>
4c*	<p>Standard column subtraction</p> $\begin{array}{r} 523 \\ - 156 \\ \hline 367 \end{array}$	
4d	<p>HTU - HTU (HT1 - HT1) with exchange e.g. 503 - 156 i.e. CANNOT take from 'next door'</p> <p>Dienes'</p>  <p>“can't take away the units so go to exchange a ten for 10 next door, BUT there isn't anything; SO go next door again (to the Hs) and exchange for TEN NEXT DOOR - SHOW the 10; still can't take away in the Is column, so go to Ts and exchange again - change 10 for 9 and take 10 next door to 1s column. NOW we can do the take away!”</p> <p>(x10)</p> <p>(x9)</p> <p>3 4 7</p>	
4d*	<p>Standard column subtraction</p> $\begin{array}{r} 503 \\ - 156 \\ \hline 347 \end{array}$	

	RULE: NEVER jump more than ONE column at a time. NEVER touch the bottom number.	
4e	<p>± NEGATIVE NUMBERS</p> <p>Line can be either vertical (as a thermometer) or horizontal (as a numberline) e.g. $\overset{-}{2} + 6 = 4$ NOTE: negative sign SHOULD be high up (superscript) to indicate position, not to confuse with minus sign (an operator). It SHOULD be read as NEGATIVE 2 not minus 2. But as weather forecasters constantly say 'minus 2' and have the sign in the centre, we follow the latter using the children's experience.</p>	<p>integer, positive, negative below zero</p> <p>consecutive numbers</p>
	Apply all the above to MONEY PROBLEMS £7.85 ± £4.20	decimal point, decimal place (with respect to money)
4f	<p>ALL CHILDREN NOW HAVE STANDARD WRITTEN METHODS FOR + AND -, compacted from the expanded methods used so far.</p> <p>More abler children move on as they are ready, including to Th H T 1 ± Th H T 1 with c.f and decomposition</p> $\begin{array}{r} 76 \\ +47 \\ \hline 123 \\ \underline{1} \end{array}$ $\begin{array}{r} 426 \\ + 395 \\ \hline 821 \\ \underline{11} \end{array}$ $\begin{array}{r} 1258 \\ + 1363 \\ \hline 2621 \\ \underline{11} \end{array}$ $\begin{array}{r} 61 \\ -47 \\ \hline 14 \end{array}$ $\begin{array}{r} 321 \\ - 167 \\ \hline 154 \end{array}$ $\begin{array}{r} 302 \\ - 166 \\ \hline 136 \end{array}$ $\begin{array}{r} 5004 \\ - 1235 \\ \hline 3769 \end{array}$	