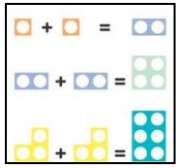
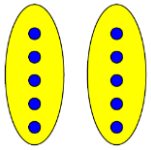
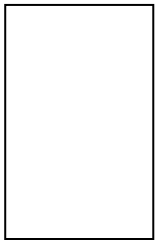
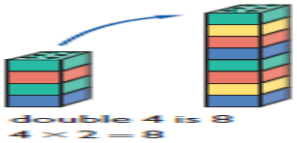
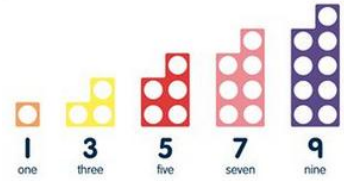
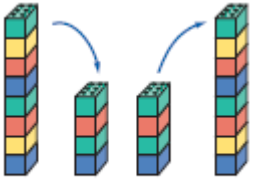
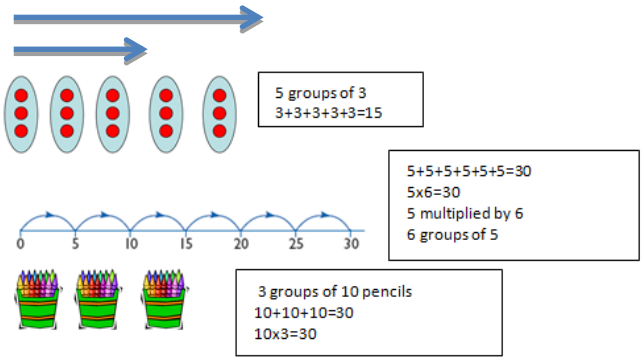
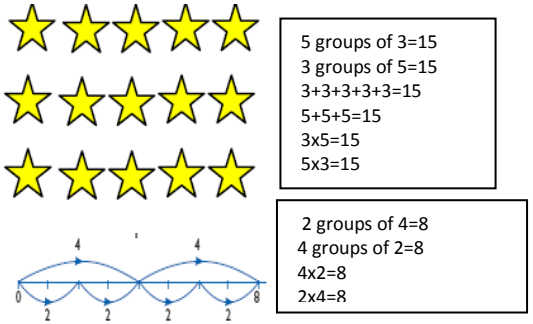
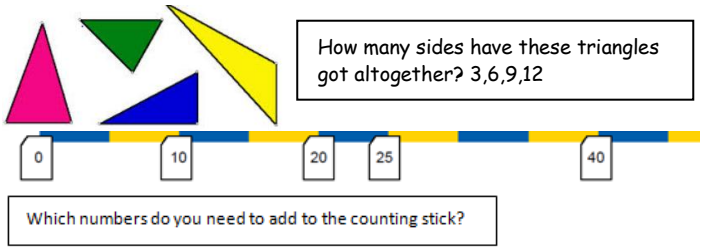
MULTIPLICATION: Y1

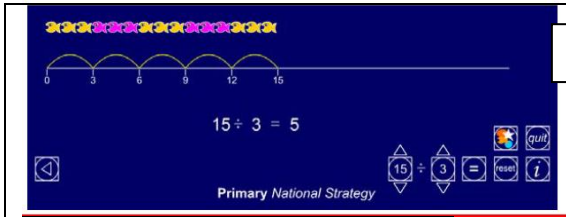
Understanding the operation and related vocabulary	Mental Calculations	Written Calculations
Understanding the operation Begin to understand multiplication by using concrete objects, pictorial representations and arrays to solve problems; make connections between the different representations. begin to use the vocabulary involved in multiplying **Vocabulary** ones, groups, lots of, doubling repeated addition array, row, column, groups of, lots of, times, columns, rows longer, bigger, higher etc times as (big, long, wide …etc) **Generalisations** Understand 6 counters can be arranged as 3+3 or 2+2+2 Understand that when counting in twos, the numbers are always even. **Some Key Questions** Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Number facts** Count in multiples of twos, fives and tens 0 2 4 6 8 10…  How many legs have 5 teddies got altogether?    How much money have I got in my purse? 5,10,15,20  How many 10ps do I need to buy a chocolate bar for 30p? Know doubles of all numbers to 10  Double 3 is ☐ 8+8=☐ Double 5 is ☐ 6+6=☐   	No formal written layout. Children will be recording their mathematics using pictorial representations, arrays, number lines and mathematical statements.  

	Begin to recognise odd and even numbers Use cubes to make 9 and recognise it is odd (as the cubes cannot be paired) Sort Numicon into odd and eve numbers  What happens if we out two odd numbers together? **Mental Methods and jottings** **Counting** Count a set of objects by grouping in 2s, 5s or 10s Count these marbles (2 at a time)  Solve problems involving doubling and equal groups I need 5 eggs to bake a cake. How many eggs will I need to bake 2 cakes? **Counting on** There are 3 pots. Each pot has 2 seeds in. How many seeds are planted? (by counting on in twos using objects or pictures to keep track) **Doubling and halving** A ladybird has 6 spots on each wing. How many spots are there altogether? (by recognising 6+6=12) 	

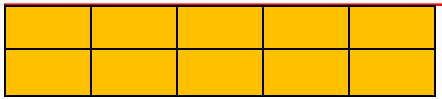
	 half of 8 is 4 8 ÷ 2 = 4 double 4 is 8 4 × 2 = 8	

MULTIPLICATION: Y2

Understanding the operation and related vocabulary	Mental Calculations	Written Calculations
Understanding the operation Understand multiplication as • repeated addition • describing an array • scaling (to compare 2 items) e.g. twice as long • correspondence problems – one to many  Show that multiplication of two numbers can be done in any order recognise that 5 x 3 is equal to 3 x 5  Recognise the inverse relationship between multiplication and division	**Number facts** Count in steps of 2, 3, and 5 from 0 0 3 6 9 12 15 18 .....30 50 45 40 35 30 ..... 0  Recall doubles of all numbers to 15 and doubles of multiples of 5 to 50 Double 13 is ☐ 11+11=☐ Double 25 is ☐ 45+45=☐ Recall and use multiplication facts for the 2, 5 and 10 multiplication tables 3 groups of 10 multiply 7 by 2 5 multiplied by 4 Recognise odd and even numbers Explain why 27 is an odd number **Mental Methods and Jottings** Calculate mathematical statements for multiplication within the multiplication tables 3x5= 7x☐ = 14 4X5= ☐ x2 **Counting on** 7x5 (by counting on in fives using fingers to keep track)	No formal written layout. Children will be recording their mathematics using pictorial representations, arrays, number lines and mathematical statements. *See jottings in other columns.* .



ITP Grouping



Write the related number sentences
5x2=10 2x5=10 10=5x2 10=2x5
10÷2=5 10÷5=2 2=10÷5 5=10÷2

Write mathematical statements using the multiplication (×), and equals (=) signs
5x4=20 16=8x2
3x□=15 □=2x7 20=□x□

Vocabulary
multiple, multiply, multiplication array, multiplication tables / facts, groups of, lots of, times, columns, rows, once, twice, three, ten…times a big, repeated addition

Generalisation
Repeated addition can be shown mentally on a number line

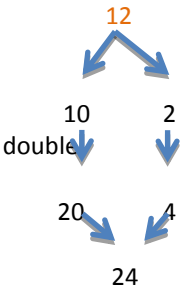
Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

With jottings 3x5 (by counting on in threes using a number line to keep track)

Doubling and halving
7x2 (by recalling the doubles fact)

With jotting
12x2 (by doubling 10, doubling 2 and recombining)



Estimating and Checking
Begin to use equivalent calculations to check answers

MULTIPLICATION: Y3

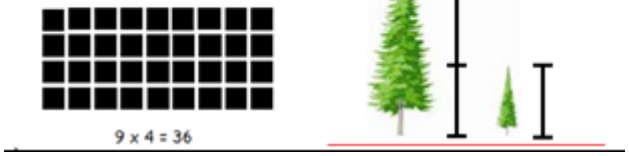
Understanding the operation and related vocabulary

Understanding the operation

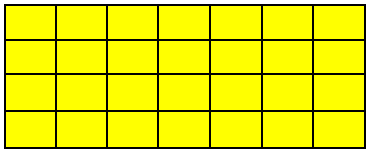
Understand multiplication as

- repeated addition
- describing an array
- scaling – comparison and enlargement
- correspondence problems – one to many and many-to-many

9+9+9+9=36
4+4+4+4+4+4+4+4+4=36



Understand commutativity and associativity
recognise that 7x4 is equal to 4x7



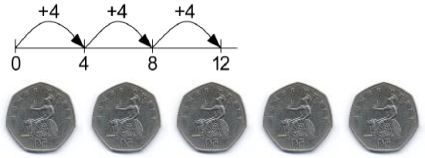
recognise that if calculating 2x3x10 the numbers can be combined in any order

Understand the inverse relationship between multiplication and division
write the related number sentences
6x3=18 3x6=18 18=6x3 18=3x6
18÷3=6 18÷6=3 3=18÷6 6=18÷3

Mental Calculations

Number facts

Count from 0 in multiples of 4, 8, 50 and 100
0 8 16 24 32 ….
500 450 400 350 ….



50,100,150,200, 250

Recall doubles of all numbers to 20, doubles of multiples of 5 to 100 and doubles of multiples of 100 to 500
Double 17 is ☐ 19x2=☐
Double 65 is ☐ 85x2=☐
Double 300 is ☐ 400+400=☐

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables and begin to use knowledge of place value to derive related facts

3 groups of 8 multiply 9 by 4
the product of 8 and 4 50 multiplied by 4

Th	H	T	U
		5	3
5	3	0	0

Place value cards
Use digit cards to make numbers in the grid.
Show how each digit in a number moves one column to the left when a number is multiples by 10 and two columns to the left when a number is multiplied by 100.

Mental Methods and Jottings

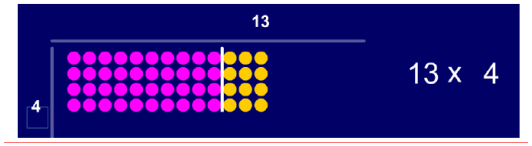
Calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers

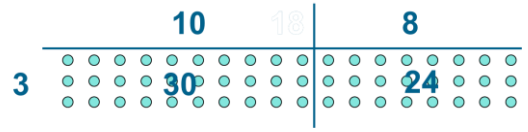
Written Calculations

Begin to use formal written methods for two-digit numbers multiplied by one-digit numbers (
multiplication facts)

ITP Multiplication Array

Use models and images to demonstrate grid method





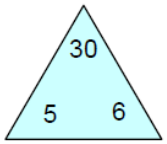
Give children opportunities for children to explore this and deepen understanding using manipulatives such as Dienes apparatus and place value counters

38x5

x	30	8
5	150	40

190

Children to use grid method by the end of year 3



$5 \times 6 = 30$
$6 \times 5 = 30$
$30 \div 6 = 5$
$30 \div 5 = 6$

Trio cards

Solve missing numbers problems involving multiplication

3x☐=15 ☐=2x7 20=☐x☐
25 + 10 = 5 x☐ 15<☐ x 2☐x☐> 20

Vocabulary
partition, grid method, inverse, product

Generalisations
Connecting x2, x4 and x8 through multiplication facts

Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this.

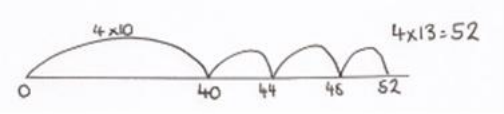
When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Use doubling to connect 2, 4 and 8 multiplication tables

Counting on
5x14 (by counting on in fives from 50)

with jottings
4x13 (by counting on in fours from 4x10 using a number line to keep track)



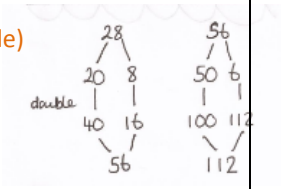
Partioning (with distributive law)
Without crossing the tens boundary
32x3= (30x3=90, 2x3=6, 90+6=96)

with jottings
Crossing the tens boundary
17x5= (10x5=50, 7x5=35, 50+35=85)

Doubling and halving

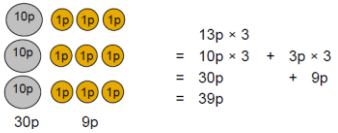
9x20 (multiply by 10 and then double)
9x10=90 Double 90 is 180

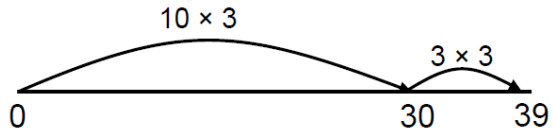
with jottings
28x4 (double and double again)
Double 28 is 56, double 56 is 112



Using known facts and place value
Use manipulatives to demonstrate this.
4x11
4x10=40 so 4x11=44

13x3





30x5
3x5=15 so 30x5=150

Estimating and checking
Estimate the answer to a calculation

38x5 is approximately 40x4

Use inverse operations and equivalent calculations to check answers.
28x4 by doubling (28x2x2) or using partitioning (20x4 and 8x4)

MULTIPLICATION: Y4

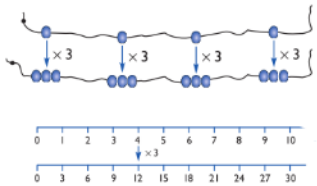
Understanding the operation and related vocabulary

Understanding the operation

Continue to understand multiplication as

- repeated addition
- describing an array
- scaling – comparison and enlargement
- correspondence problems – one to many and many-to-many

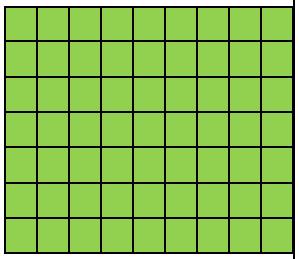
11+11+11+11=44
7+7+7+7+7+7+7=49



Understand the distributive law

recognise that 14x5 is the same as 10x5 added to 4x5

19 x 5 = (9 x 5) + (10 x 5) = 45 + 50 = 95
36 x 9 = (30 x 9) + (6 x 9) = 270 + 54 = 324



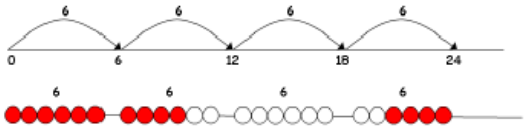
continue to understand commutativity and associativity
recognise that 7x9 is equal to 9x7
recognise that if calculating 4x8x10 the numbers can be combined in any order
4 x 8 x 10 = 320
8 x 10 x 4 = 320
10 x 4 x 8 = 320

Continue to understand the inverse relationship between multiplication and division
write the related number sentences

Mental Calculations

Number facts

Count in multiples of 6, 7, 9, 25 and 1000
0 7 14 21 28 …
300 275 250 225 200 …



25	50			
100				
	325		375	

ITP Number Grid

Derive doubles of multiples of 50 to 1000 and multiples of 1000
Double 950 is ☐ 750x2=☐
Double 8000 is ☐ 6000+6000=☐

Recall multiplication facts for multiplication tables up to 12 × 12, and use place value to derive related facts

7 groups of 8 multiply 9 by 6
the product of 8 and 11 60 multiplied by 4

Recognise factor pairs
list the factors pairs of 32

Mental Methods and Jottings

Multiply mentally using place value, known and derived facts, including: multiplying by 0 and 1; multiplying together three numbers

Written Calculations

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Children to embed and deepen their understanding of the grid method to multiply. Ensure this is still linked back to their understanding of arrays and place value counters.

36 x 4 = 144

X	30	6
4	120	24

120 + 24 = 144 (add the partial products)

127 x 6 = 762

x	100	20	7
6	600	120	42

600 + 120 + 42 = 762 (add the partial products)

6x7=42 7x6=42 42=6x7 42=7x6
42÷7=6 42÷6=7 7=42÷6 6=42÷7

Solve missing numbers problems involving multiplication

3x□=15 □=2x7 20=□x□
25 + 10 = 5 x□ 15<□ x 2□x□> 20

Vocabulary
Factor

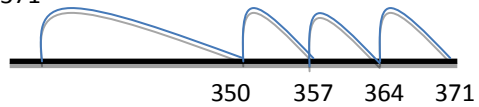
Generalisations
When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Counting on
3x42 (by counting on in threes from 120)

With jottings
7x53 (by counting on in sevens from 7x50 using a number line to keep track)

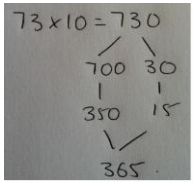
3 X 50
3 X 53 = 371



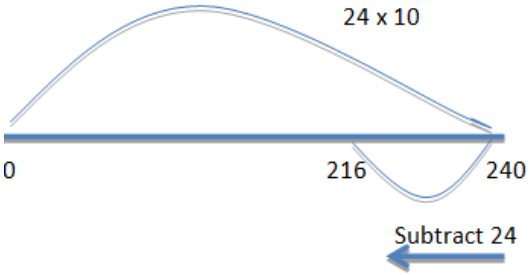
Partitioning (using the distributive law)
53x6 (50x6=300 3x6=18 300+18=318)
with jottings
86x7 (80x7=560 6x7=42 560+42)

Using doubling and halving

35x8 (double, double and double again)
Double 35 is 70, double 70 is 140, double 140 is 280
With jottings
73x5 (multiply by 10 and then halve)
73x10=730 Half of 730 is 365 (Some children may need to partition 730 in a different way)



Using factors
15x6 = 15x3x2
15x3=45 45x2=90
with jottings
8x18 = 8x9x2

	8x9=72 72x2=144 **<u>Using known facts and place value</u>** 24x10=240 so 24x9=216 (by subtracting 24 from 240) 800x6 8x6=48 so 800x6=4800  	

MULTIPLICATION: Y5

Understanding the operation and related vocabulary

Understanding the operation

Continue to develop understanding of multiplication to include:

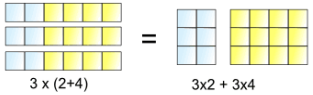
- scaling by simple fractions
- simple rates

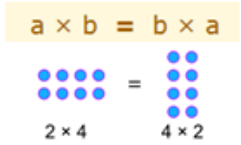
Continue to understand the distributive, commutative and associative laws

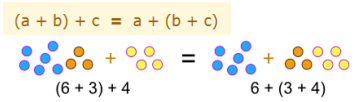
recognise that 37x6 is the same as 30x6 added to 7x6 (distributive)

recognise that 25x7 is equal to 7x25 (commutative)

recognise that if calculating 18x4x10 the numbers can be combined in any order (associative)







Continue to understand the inverse relationship between multiplication and division

Mental Calculations

Number facts

Use knowledge of counting in multiples to count in decimal steps (one decimal place)

0.6 1.2 1.8 2.4 ….

8.4 7.7 7.0 6.3 …

Derive doubles of decimals (to one decimal place) using knowledge of place value

Double 0.4 is ☐ 0.7x2=☐

Double 3.8 is ☐ 5.6+5.6=☐

Continue to recall multiplication facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts

7 groups of 8 multiply 12 by 9

the product of 80 and 40 0.6 multiplied by 4

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Identify multiples and factors, and common factors of two numbers.

list the factors of 96

identify the common factors of 30 and 36 by listing factor pairs

give a number that is a multiple of 3 and a multiple of 2 (and recognise these are multiples of 6)

list the multiples of 9 between 150 and 180 (using tests of divisibility)

Written Calculations

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Multiply numbers with up to one decimal place by one-digit whole number.

Use grid method, progressing to short and long multiplication for numbers with more digits only when children have secure understanding of multiplication facts and place value. Continue to embed understanding through the use of manipulatives and grid method.

Grid method

46 x 82 43.2 x 7

×	30	5
20	600	100
6	180	30

600 + 100 = 700
180 + 30 = 210
700 + 210 = 910

x	6
2.0	12.0
0.3	1.8

13.8

Short multiplication

36 x 4 = 144

30 + 6
X 4
24 (4 x 6 = 24)
+120 (4 x 30 = 120)
144

Include an addition symbol when adding partial products.

36x4= 36
X 4
144
2

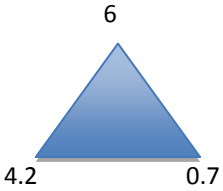
Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28

327
X 4
1308
1 2

Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

write the related number sentences
6x0.7=4.2 0.7x6=4.2 4.2=6x0.7 4.2=0.7x6
4.2÷0.7=6 4.2÷6=0.7 0.7=4.2÷6 6=4.2÷0.7



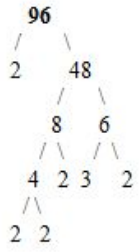
Continue to solve missing number problems
6x☐=540 ☐=0.4x8 480=☐x☐
90x40 = 6x☐ 2.5<☐ x5 ☐x☐> 700x8

begin to use brackets
(10+3) x 7 = ☐ ☐=10 + (0.4 x 8)

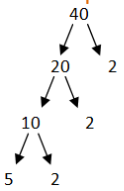
Vocabulary
cube numbers
prime numbers
square numbers
common factors
prime number, prime factors
composite numbers

Generalisation
Relating arrays to an understanding of square numbers and making cubes to show cube numbers.
Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?
How do you know this is a prime number?



Establish whether a number up to 100 is prime and recall primes up to 19; find prime factors
explain why 23 is a prime number
list the prime factors of 20 (20 = 2x10 = 2x2x5)



5 x 2 x 2 x 2

Recognise and use square and cube numbers
What is… 8^2? 3^3?

Mental Methods and Jottings

Multiply numbers mentally drawing upon known facts

use factors to construct equivalence statements

4 x 35 = 2 x 2 x 35
3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10

begin to multiply tenths, and one-digit whole numbers and tenths by one-digit whole numbers

0.2 x 3 = 0.6

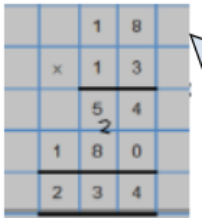
Partitioning (using the distributive law)
1.2x7 (1x7=7 0.2x7=1.4 7+1.4=8.4)
With jottings
3.5x7 (3x7=21 0.5x7=3.5 21+3.5=24.5)

Long multiplication – expanded method

31
X 25
5 (5x1)
150(5x30)
20(20x1)
600(20x30)
775

Long multiplication

3 1	Initially ,some children may need to break this down further (See NCETM website)		
x2 5			
1 5 5	31	31	155
6 2 0	x 5	x 20	+ 620
7 7 5	15 5	620	775



	Doubling and halving 3.7x4 (Double and double again) Double 3.7 is 7.4, double 7.4 is 14.8 with jottings 76x50 (multiply by 100 and halve) 76x100=7600 Half of 7600 is 3800 **Using factors** 25x12=25x2x6 25x2=50 50x6=300 with jottings 3x270=3x3x9x10 3x3x9=$9^2$$9^2$=81 81x10=810 **Using Known facts and place value** 13x19 13x20=260 so 13x19=247 (subtract 26 from 260) 3x14 recognise 3x14 is equivalent to 6x7 **Estimating and Checking** Check 86x9 by using an equivalent calculation Multiply by 10 and adjust (860-86) or partition (80x9 added to 6x9)	

MULTIPLICATION: Y6

Understanding the operation and related vocabulary

Understanding the operation

Continue to understand

- Scaling by fractions
  Of the 90 students on a field trip to the zoo, two ninths want to go to see the bears. How many students want to see the bears?
  90÷ 2/9 90 ÷9 = 10 10 X 2 = 20
- Rate
  A car travels 60 miles per hour. How far will it travel in 2 and a quarter hours?

Use their knowledge of the order of operations -BODMAS
Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction
Understand that if the operations are at the same level of priority, work out the example from left to right

Continue to solve missing number problems
6x☐=0.54 ☐=0.06x8 4.8=☐x☐
0.9x4 = 6x☐ 0.63<☐ x0.09 ☐x☐>0.07x8

Explore the order of operations using brackets
compare 14 ÷ (2 x 5) with (14 ÷ 2) x 5

Vocabulary

common factor/multiple

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic to remember this or could be encouraged to design their own ways of remembering.
Understanding the use of multiplication to support conversions between units of measurement.

Mental Calculations

Number facts

Use knowledge of counting in multiples to count in decimal steps (two decimal places)
0.09 0.18 0.27 0.36 ….
0.48 0.44 0.4 0.36 …

Derive doubles of decimals (to two decimal places) using knowledge of place value
Double 0.47 is ☐ 0.73x2=☐
Double 3.08 is ☐ 2.59+2.59=☐

Continue to recall multiplication facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts
30 multiplied by 800 multiply 0.12 by 6
the product of 0.08 and 4 0.4 multiplied by 0.5

identify common factors, common multiples and prim numbers
find the highest common factor of 18 and 24
find the lowest common multiple of 6 and 15
identify whether 87 is a prime number
list the prime factors of 84 (84 = 2x42 = 2x2x21 = 2x2x3x7)
use the tests of divisibility to identify factors and multiples

continue to use square and cube numbers
What is…12^2? 6^3?

Mental Methods and Jottings

Perform mental calculations, including with mixed operations, large numbers and decimals

Written Calculations

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Multiply numbers with up to two decimal places by one-digit and two-digit whole numbers

GRID METHOD

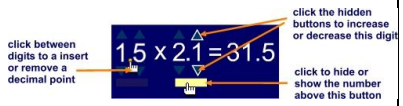
15.76 x 3

E.g. 15.76 × 3 =

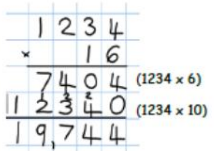
X	10.00	5.00	0.70	0.06
3	30.00	15.00	2.10	0.18

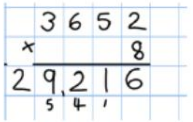
30.00
15.00
2.10
0.18
+
47.28

ITP multiplication grid



Children should not be taught the following method shown below until they are thoroughly secure with mental calculation strategies, recall of multiplication tables and the application of Place Value (see year 5)
Develop year 5 methods with more complex calculations such as





Some Key Questions What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Partitioning (using distributive law)** 6.04x3 (6x3=18 0.04x3=0.12 18+0.12=18.12) With jottings 0.43x6 (0.4x6=2.4 0.03x6=0.18 2.4+0.18=2.58) **Doubling and halving** 0.24x40 (double and double again, then multiply by 10) Double 0.24 is 0.48, double 0.48 is 0.96, 0.96x10=9.6 With jottings 68x25 (multiply by 100, then halve and halve again) 68x100=6800 Half of 6800 is 3400 Half of 3400 is 1700 **Using factors** 1.5x16=1.5x2x8 1.5x2=3 3x8=24 with jottings 32x24 = 32x3x8 32x3=96 96x8=800-(4x8)=768 **Using known facts and Place value** 17x98 17x100=1700 so 17x98 is 1666 (subtract 17x2 from 1700) 15x18 recognise 15x18 is equivalent to 30x9 **Estimating and checking** Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. 5872x54 is approximately 6000x50 Continue to use appropriate strategies to check answers Check 496x5 by using an equivalent calculation Multiply by 10 and halve or use a known fact and adjust (500x5) –(4x5)	