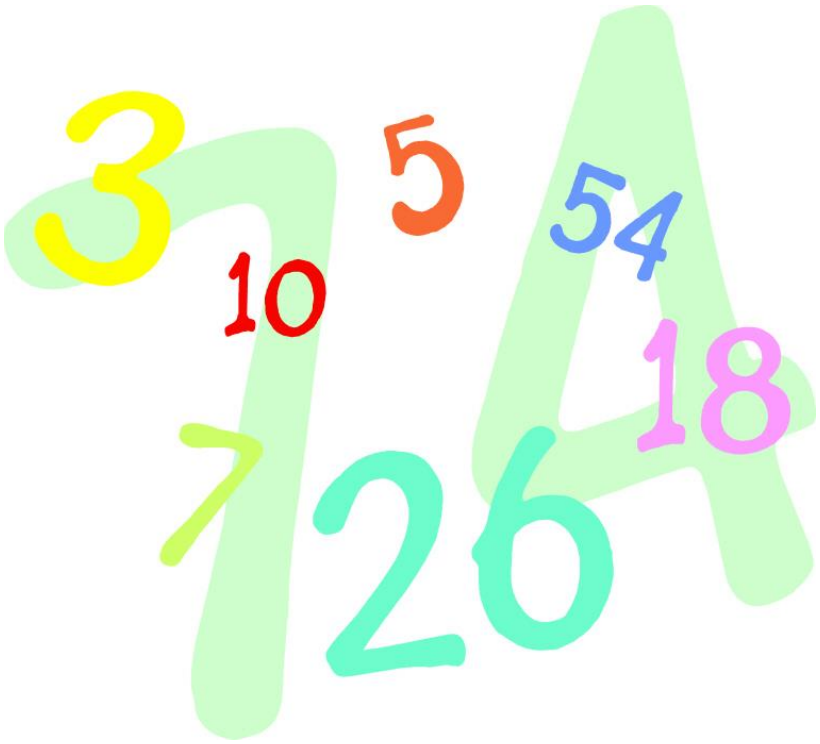
Helping your child with number calculations



Hartford Junior School

April 2015

Number Calculations

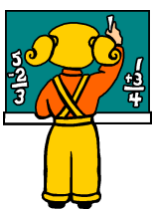
The maths work your child carries out in school may look a lot different from the kind of 'sums' you remember doing! This is because today children are encouraged to work mentally, where possible, using jottings to help support their thinking. Even when children are taught more formal written methods, they are only encouraged to use them for calculations they cannot solve mentally.

You will hear your child talking about a range of different methods, which may sound confusing to you. Methods such as **number lines, partitioning, arrays** and **chunking** are being used commonly throughout school. These new ways of working give your child a greater **understanding** of number and the number system.

Today, a large part of your child's work in maths is to explain their reasoning and to discuss the suitability of different strategies. Let your child tell you how they solved a calculation and why they used a particular strategy.

In class, children will explore these approaches using a wide range of resources to ensure they understand the mathematics behind the methods. This then avoids 'rote' learning like some of us experienced when we were at school. Our primary aim is that children have a solid understanding of the methods they are using.

This booklet will help you understand the methods your child is learning in school and will help you support them at home. Please remember that not all children in one year group will be working on the same stage. Some children will be more able and will be working on a stage above, just as some will need more reinforcement and be working on a previous stage. If you are unsure of which stage your child is using, please ask their class teacher.



What can you do to help your child with their calculations?

It is important to talk to your child about their calculations and how they have solved them. These questions will help you support your child's thinking when solving calculations.

When faced with a calculation problem encourage your child to ask...

- Can I do this in my head?
- Which jottings would help me?
- Would a number line help me?
- Do I need to use a written method?
- Do I need a calculator? (Year 5/6)

Also encourage your child to round the numbers up or down to help them estimate an answer before solving the calculation.

Encourage them to ask...

- Is the answer close to the estimate?
- Does the answer make sense?
- Is the answer reasonable?

Over the next pages you will find a stage by stage progression for each of the different number operations. Whatever stage your child is on, they may recap on previous stages as the numbers become more complex, for instance when introducing decimals.

+ Addition +

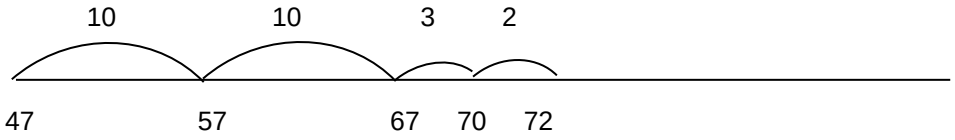
Stage 1

Empty Number Line

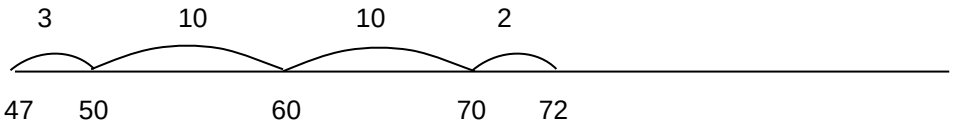
Using an empty number line to count on (Jumping on in groups of numbers is much more efficient than counting on in ones)

E.g. 47 + 25 = 72

Counting on tens first then units



Counting on to the nearest ten, then in tens and the remaining units



As children become more confident they will jump on in multiples of ten and not break the units down when crossing tens boundaries.

Stage 2

Partitioning

Partitioning means to split a number into two parts. For most addition we split the numbers into hundreds, tens and units then recombine them to find the answer.

E.g. 47 + 56 = 103

40 + 7 + 50 + 6
40 + 50 = 90
7 + 6 = 13
90 + 13 = 103

E.g. 135 + 352 = 487

100 + 30 + 5 + 300 + 50 + 2
100 + 300 = 400
30 + 50 = 80
5 + 2 = 7
400 + 80 + 7 = 487

Doubling numbers

Double 34 = 68
30 + 30 + 4 + 4
30 + 30 = 60
4 + 4 = 8
60 + 8 = 68

Stage 3

Expanded Column Method

This method builds on your child's understanding of partitioning and makes the value of the digits clear. The language used is very important and the children use words such as hundreds, tens and units.

Start by partitioning the numbers into hundreds, tens and units. Then add them together, starting with the lowest numbers (units, tens, hundreds)

E.g. 74 + 23 = 97

70 + 4
20 + 3
90 + 7 = 97

E.g. 287 + 541 = 828

200 + 80 + 7
500 + 40 + 1
700 +120 + 8 = 828

Stage 4

Expanded Method

The expanded method progresses in difficulty and is set out more in columns. We always add the units before the tens, then the hundreds, as this sets up good practise for using the compact method in Stage 5. It is important to remind the children of the value of the tens digits, so for the question below you would refer to the 7 in 76 as 70 and the 4 in the 47 as 40

E.g. 76 + 47 = 123

```
   7  6
+  4  7
------
   1  3   ← Add the units first (6+7)
1  1  0   ← Then add the tens (70 + 40)
------
1  2  3   ← Add the tens and units together to find the answer
```

E.g. 251 + 385 = 636

```
   2  5  1
+  3  8  5
---------
         6
   1  3  0
   5  0  0   ← Remember to add the hundreds numbers together
---------
   6  3  6
```

Stage 5

Compact Method

This method is probably the method you are familiar with from your school days. Once the children have developed a solid understanding of the value of the digits in numerals this method is the most efficient. This method is used when adding numbers with many digits and also with decimals.

E.g. 234 + 348 = 582

```
   2  3  4
+  3  4  8   ← Start by adding the units first, then the tens and the hundreds
---------
   5 ¹8  2   ← If the answer crosses the tens boundary then we carry the ten in the next column in the space where the answer will go, so the children won't forget to add it on
```

E.g. 2.63 + 17.15 = 19.78

```
   2  .  6  3
+1 7  .  1  5
------------
  1  9  .  7  8   ← Remember the decimal point stays in the same place
```

- Subtraction -

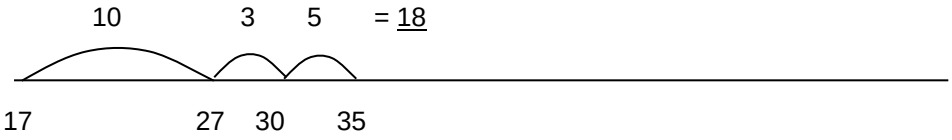
Stage 1

Empty Number Line

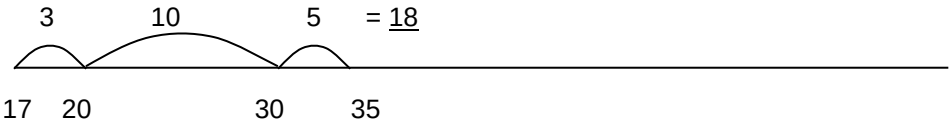
Using an empty number line for subtraction can be carried out in a variety of different ways, both counting forwards and backwards.

E.g. Find the difference between 17 and 35 or 35 – 17 = 18

Counting on tens first then units

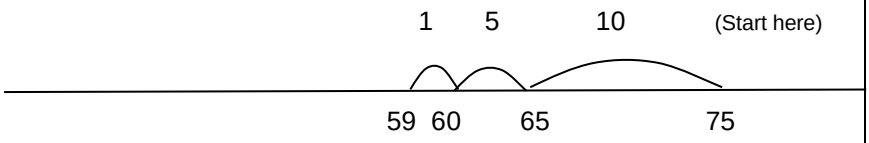


Count on from the smaller to larger number (to the nearest 10 first)



E.g. 75 – 16 = 59

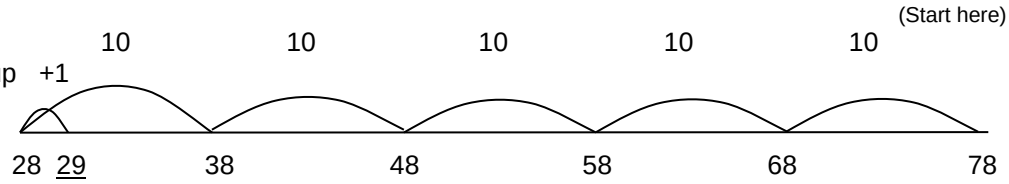
Counting back to the smaller number from the larger one



Counting back near multiples of 10 then adjusting by one or two as necessary.

E.g. 78 – 49 = 29

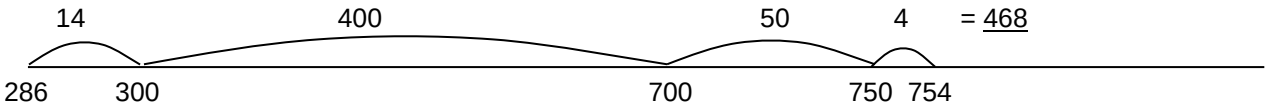
Round the lower of the digits up to a tens number



Finding the Difference on a Number Line

This involves 'counting on' from the smaller number to the larger one which is called 'finding he difference' (another model of subtraction).

E.g. 754 – 286 = 468



Start with the lower number. Make jumps to 'sensible' multiples of 10 or 100. Once the target number is reached, add up all the amounts that have been added on.

Stage 2

Expanded Decomposition

This involves partitioning of numbers to subtract. With subtraction, as with addition, we can partition numbers into hundreds, tens and units. Numbers will be chosen carefully so that exchanging is not required to begin with. The children with then be introduced to calculations where it will be necessary to exchange digits across the columns.

E.g. 86 – 34 = 52

Exchanging digits across columns not necessary.

$$\begin{array}{rrrrrr} & 8 & 0 & + & 6 & \\ - & 3 & 0 & + & 4 & \\ \hline & 5 & 0 & + & 2 & = 52 \end{array}$$

E.g. 74 – 27 = 47

Exchanging digits across columns needed.

$$\begin{array}{rrrrrr} & \overset{6}{\cancel{7}} & \overset{0}{\cancel{0}} & + & {}^{1}4 & \\ - & 2 & 0 & + & 7 & \\ \hline & 4 & 0 & + & 7 & = 47 \end{array}$$

One of the tens has been exchanged for 10 units giving 60 tens and 14 units.

Stage 3

Compact Decomposition

This method is similar to the compact method for addition and involves exchanging tens for units etc. This method can also be used when subtracting decimals.

E.g 132 – 74 = 58

One ten has been exchanged into units leaving 2 tens.

Then one hundred has been exchanged into tens.

$$\begin{array}{rrrr} & {}^{1}\cancel{1} & {}^{1}\cancel{3}^{2} & {}^{1}2 \\ - & & 7 & 4 \\ \hline & & 5 & 8 \end{array}$$

E.g. 6467 – 2684 = 3783

One thousand has been exchanged into hundreds leaving 5 thousands

One hundred has been exchanged into tens leaving 3 hundreds

$$\begin{array}{rrrrr} & {}^{5}\cancel{6} & {}^{13}\cancel{4} & {}^{1}6 & 7 \\ - & 2 & 6 & 8 & 4 \\ \hline & 3 & 7 & 8 & 3 \end{array}$$

Choosing the best method

Although these methods are taught in the order above, the children are expected to continue to use them all. With subtraction, the compact written method is not always the best method to use. This is especially the case if a lot of exchanging is required or if the larger number contains a lot of 0 digits. For this is it probably best completed counting up on a number line or mentally with a number line and jottings to assist them.

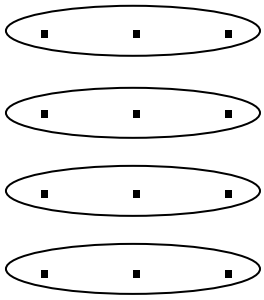
x Multiplication x

Stage 1

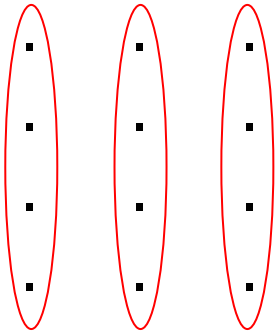
Arrays

When learning to multiply and divide arrays are a very visual and easy way for children to group objects to help them solve simple calculations. They group objects into rows and columns. It helps children to understand that the order of the numerals in a multiplication question are interchangeable (e.g. 4 x 3 = 3 x 4) and it gives them a picture to relate to a number.

E.g. 3 x 4 = 12



This shows 4 rows of 3.



It also shows 3 columns of 4.

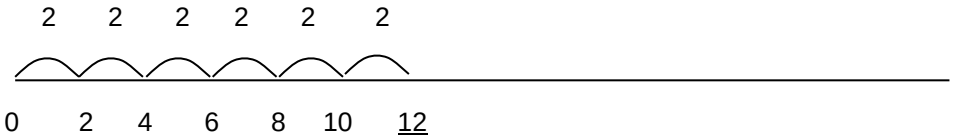
Stage 2

Empty Number Line

With Multiplication an empty number line can help children count in groups of a particular number. This is called Repeated Addition as the same number is repeatedly added.

E.g. 2 x 6 = 12

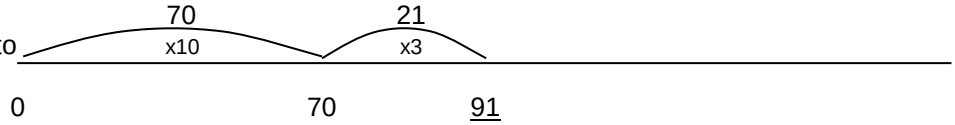
Count on 6 jumps of 2



When the children become more confident they will not count every jump, but group them for ease.

E.g. 7 x 13 = 91

Partition the second number into tens and units



Stage 3

Grid Method

The grid method is an easy method for solving multiplication questions when children have learnt all of their basic times tables. If they do not know their multiplication tables off by heart they can find this method difficult. They also need to have a good working knowledge of how to partition numbers into tens and units. It can also be used with decimals.

E.g. 38 x 7 = 266

x	30	8
7	210	56

=266

← First partition the 2 digit number

←Multiply the partitioned numbers by 7

← Then finally add the two answers together

E.g. 372 x 24 (We know it will be nearly 400 x 20 = 8000) = 8928

x	300	70	2	
20	6000	1400	40	= 7440
4	1200	280	8	= 1488
				8928

NB This is our accepted formal written method. Many children will be confident in using this method and continue to use it throughout the school.

Stage 4

Expanded Short Method

As with addition we start by multiplying the units first.

E.g. 38 x 7 = 266

```
   3  8
x     7
-------
   5  6   ← 8 x 7
2  1  0   ← 30 x 7
-------
2  6  6
```

```
      3  7  2
x        2  4
-------------
            8
      2  8  0
1     2  0  0
         4  0
1     4  0  0
6     0  0  0
-------------
8  ¹9  2  8
```

Stage 5

Short Method

This method also involves carrying numbers in the next column, like addition. This will be the method you are most familiar with.

E.g. 38 x 7 = 266

```
    3  8
x      7
---------
  2 ⁵6  6
```

E.g. 372 x 24 = 8928

```
      3  7  2
x        2  4
-------------
   1 ²4  8  8   x 4 first
¹7   4  4  0   x 20 next
-------------
   8 ¹9  2  8
```

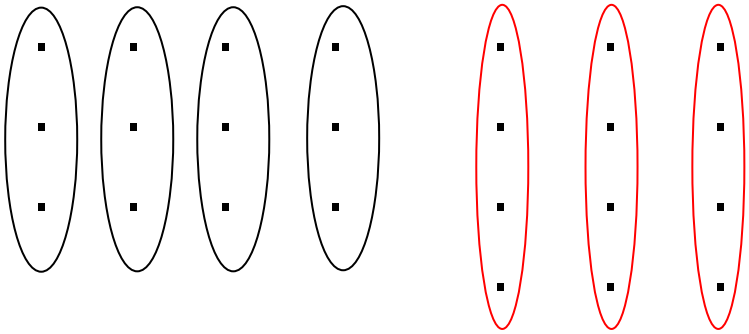
÷ Division ÷

Stage 1

Arrays

As with multiplication the children draw marks in rows or columns to find out the answer to the question. Remember to check the answer using your times tables.

E.g. 12 ÷ 3 = 4 12 ÷ 4 = 3



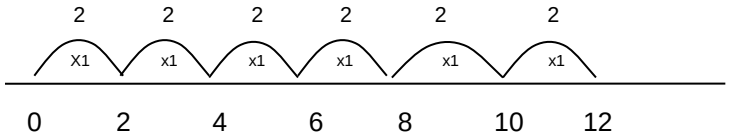
Stage 2

Empty Number Line

We use an empty number line to solve simple division questions through the Repeated Addition method. (This emphasises the links with multiplication.)

E.g. 12 ÷ 2 = 6

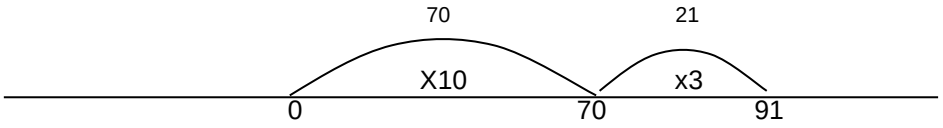
Count on in 2's until you reach the target number. In this case each jump is x1. Count up how many jumps it took.



When the children become more confident they will not count every jump but group them for ease.

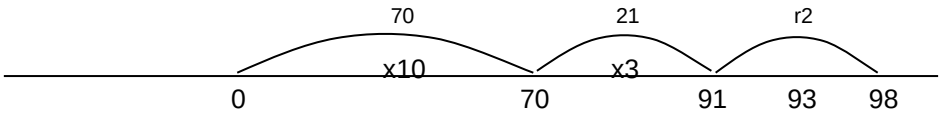
E.g. 91÷ 7 = 13

Use facts you know
Add together the 'x10' and 'x3' to find the answer



E.g. 93 ÷ 7 = 13 r2 (Use number lines to introduce the idea of remainders)

Another jump of 7 will take us past 93, so we can work out the remainder



The number line may also be presented to the children vertically. As well as jumping up, children will also be encouraged to count back, so they are subtracting 'chunks' of numbers.

Stage 3

Chunking

This method moves the children on from number lines to something more formal. It groups the jumps in the repeated subtraction method in the previous stage into chunks.

E.g. 182 ÷ 7 = 26

```
      2  6
   ┌──────
7  │ 1  8  2
-  │ 1  4  0     20
   ├──────
      4  2     ← number left when - 140
   -  4  2     6
   ├──────
                  total the number of 'chunks'
                  and write the answer at the top
```

E.g. 977 ÷ 36 = 27 r5

```
      2  7  r 5          20 + 5 + 2
   ┌──────
36 │ 9  7  7
-  │ 7  2  0     20
   ├──────
      2  5  7     ← number left when - 720
   -  1  8  0     5
   ├──────
         7  7     ← number left when -180
         7  2     2
   ├──────
            5     ← remainder
```

This may be introduced in a different way to emphasise the links between multiplication and the grid method.

E.g. 96 ÷ 6 = 16
The same grid is drawn as though they are going to multiply. However, the middle of the grid is filled in (the number to be divided up) and the number by which it is to be divided is placed down the side.

The children use their multiples knowledge to partition 96 into the 'chunks'. In this case, into 60, 30 and 6 (which add up to 96) and put these in the middle of the grid.

The children then work out how many multiples of 6 each section is made up from and is then able to complete the grid along the top. These numbers are then added to find the answer.

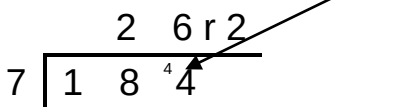
x	?	?	?
6	60	30	6

Stage 4

Short Division

This method is used when dividing by a one digit number. It condenses the steps that are used in chunking and children exchange digits from one column to the next.

E.g. 184 ÷ 7 = 26 r2



You divide 180 by 7 twenty times leaving 40, or 4 tens , which are exchanged to 40 units and carried with the 4 units.

This answer could also be expressed as 26 2/7

Stage 5

Long Division

Long division is used when dividing by 2 digit numbers or more.

E.g. 977 ÷ 36 = 27 r5

```
        2  7  r 5
    ------------
36 | 9  7  7
     7  2  0      ← 36 goes into 720 20 times so 2 is written above the line to represent 20
    ---------
     2  5  7      ← The remaining amount is brought down and divided again.
     2  5  2      ← 36 goes into 252 7 times so 7 is written above the line to represent it
    ---------
           5      ← This 5 cannot be divided any further so it is kept as a remainder
```