

Progression Towards a Written Method for Addition

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e. $5 + 3 = 3 + 5$
- associative i.e. $5 + 3 + 7 = 5 + (3 + 7)$

The fact that it is commutative and associative means that calculations can be rearranged, e.g. $4 + 13 = 17$ is the same as $13 + 4 = 17$.

YR

Early Learning Goal:

Using quantities and objects, children add two single-digit numbers and count on to find the answer.

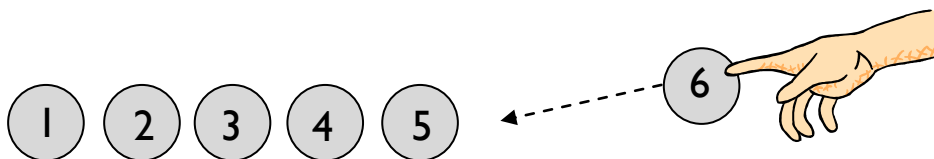
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating $4 + 2$, they are encouraged to count out four counters and count out two counters.



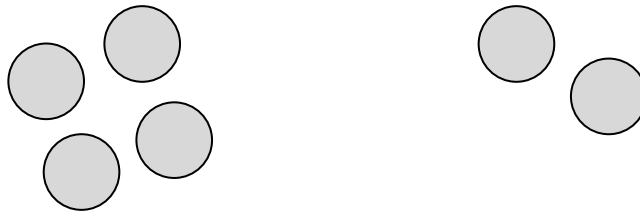
To find how many altogether, touch and drag them into a line one at a time whilst counting.



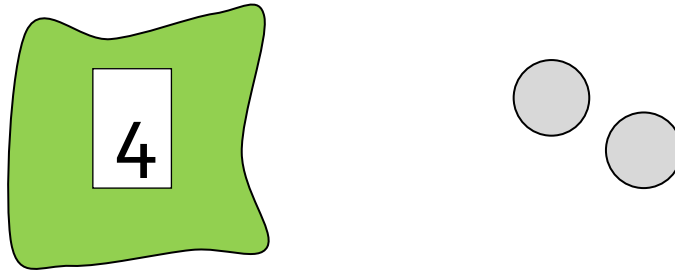
By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating $4 + 2$, count out the two groups of counters as before.



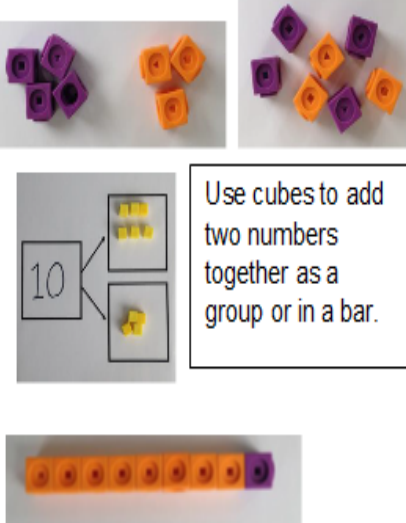
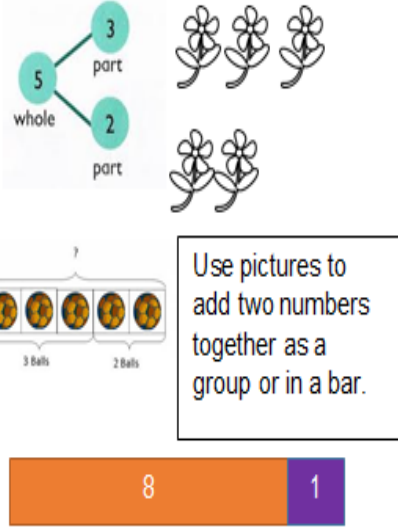

then cover up the larger group with a cloth.




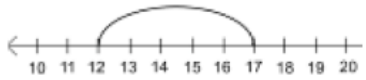
For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Those who are ready may record their own calculations.




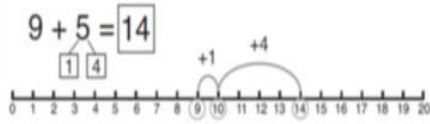
Objective :Combining two parts to make a whole: part- whole model

Concrete	Pictorial	Abstract
 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>

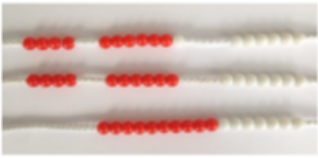
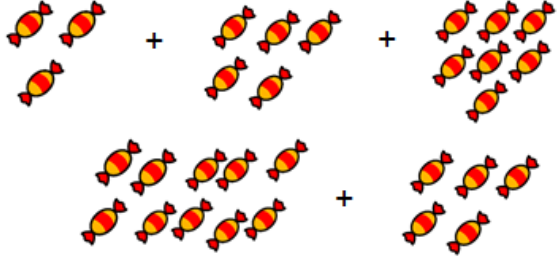
Objective: starting at the bigger number and counting on

 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
---	--	--

Objective :Regrouping to make 10

<p>Regrouping to make 10.</p>	 <p>$6 + 5 = 11$</p>  <p>Start with the bigger number and use the smaller number to make 10.</p>	 <p>$3 + 9 =$</p> <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p> <p>$9 + 5 = 14$</p> 	<p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
-------------------------------	--	---	--

Objective: Adding 3 single digit numbers

<p>$4 + 7 + 6 = 17$</p> <p>Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p>	 <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p>	<p>$4 + 7 + 6 = 10 + 7$</p> <p>$= 17$</p> <p>Combine the two numbers that make 10 and then add on the remainder.</p>
--	--	--

Y1

End of Year Objective:
Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value, they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and units. For example, when adding 11 and 5, they can make the 11 using a ten rod and a unit.



The units can then be combined to aid with seeing the final total, e.g.



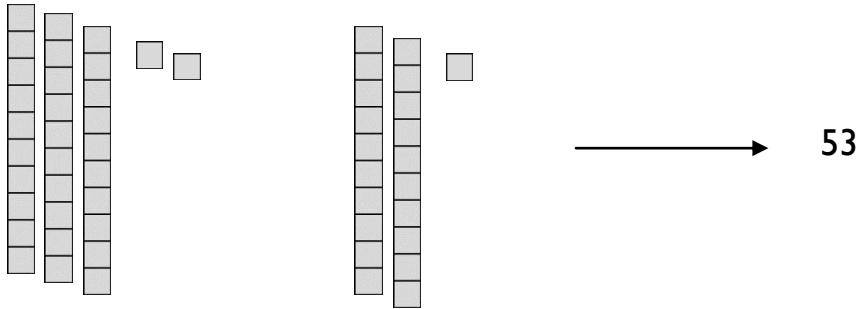
so $11 + 5 = 16$. If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

Y2

End of Year Objective:

Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.

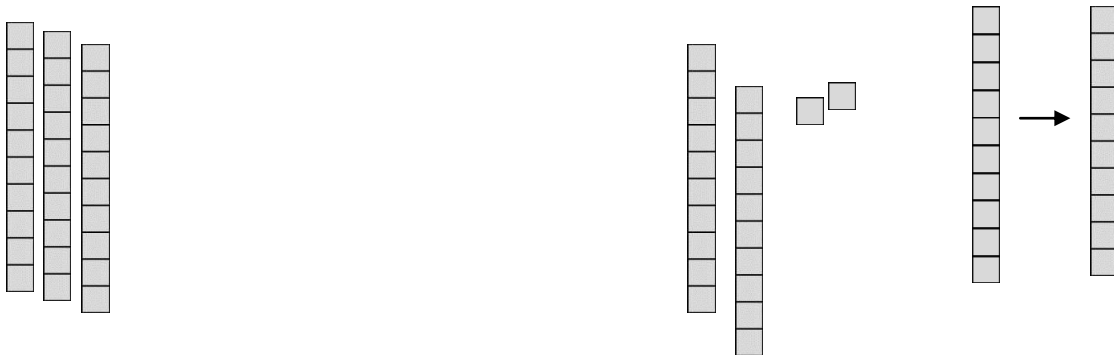
Children will continue to use the Base 10 equipment to support their calculations. For example, to calculate $32 + 21$, they can make the individual amounts, counting the tens first and then count on the units.



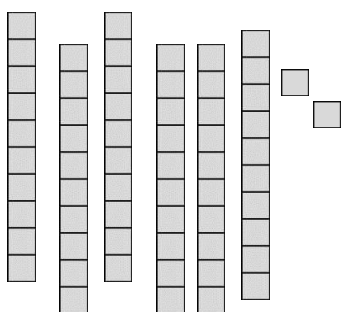
When the units total more than 10, children should be encouraged to exchange 10 units/ones for 1 ten. This is the start of children understanding ‘carrying’ in vertical addition. For example, when calculating $35 + 27$, they can represent the amounts using Base 10 as shown:



Then, identifying the fact that there are enough units/ones to exchange for a ten, they can carry out this exchange:

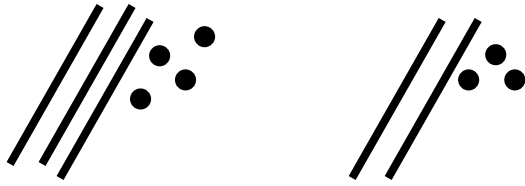


To leave:

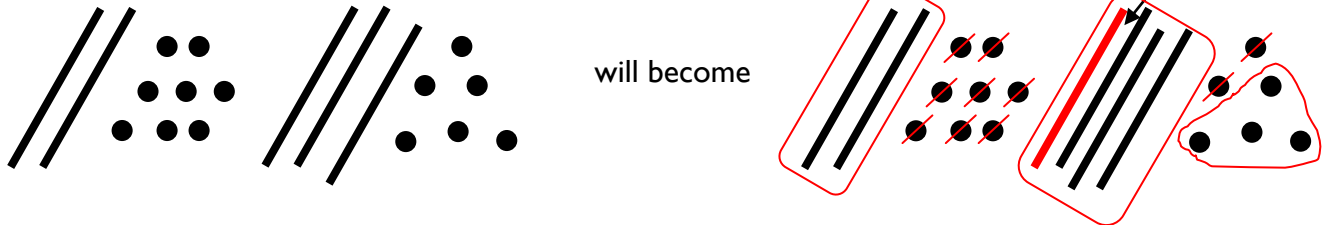


Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks).

e.g. $34 + 23 =$



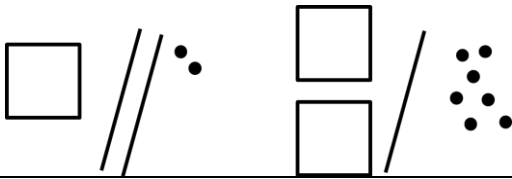
With exchange:
e.g. $28 + 36 =$



so $28 + 36 = 64$

It is important that children circle the remaining tens and units/ones after exchange to identify the amount remaining.

This method can also be used with adding three digit numbers, e.g. $122 + 217$ using a square as the representation of 100.



Column method no regrouping using place value counters (after using base 10)

<p>$24 + 15 =$ Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p>	<p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p>	<p><u>Calculations</u></p> $21 + 42 =$ $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$
--	---	--

Y3

End of Year Objective:
Add numbers with up to three digits, using formal written method of columnar addition.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

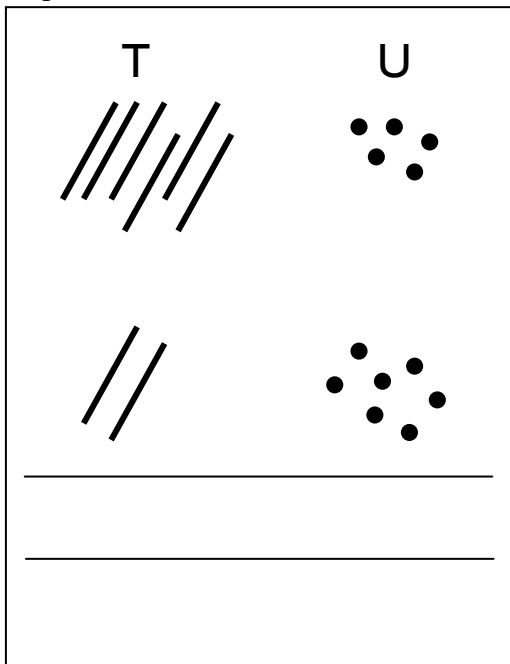
It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

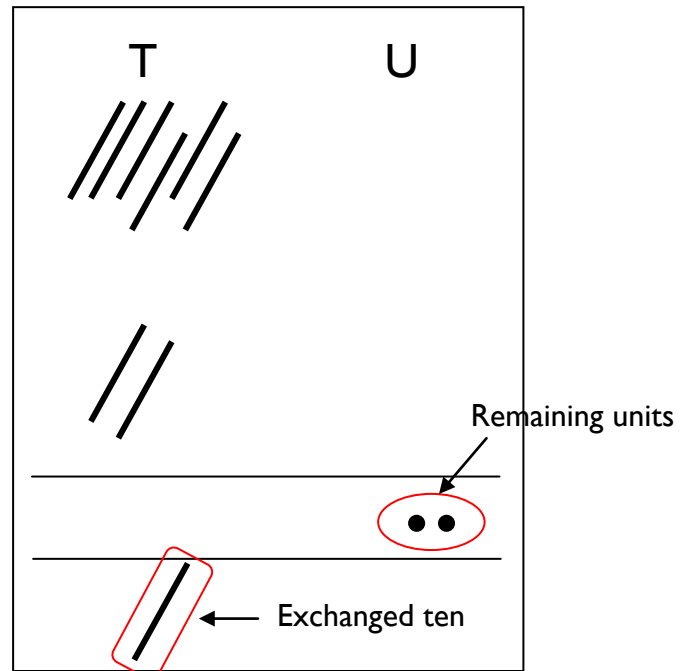
Children should add the **least significant digits** first (i.e. start with the units/ones), and in an identical method to that from year 2, should identify whether there are greater than ten units which can be exchanged for one ten.

They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below).e.g. $65 + 27$

Step 1



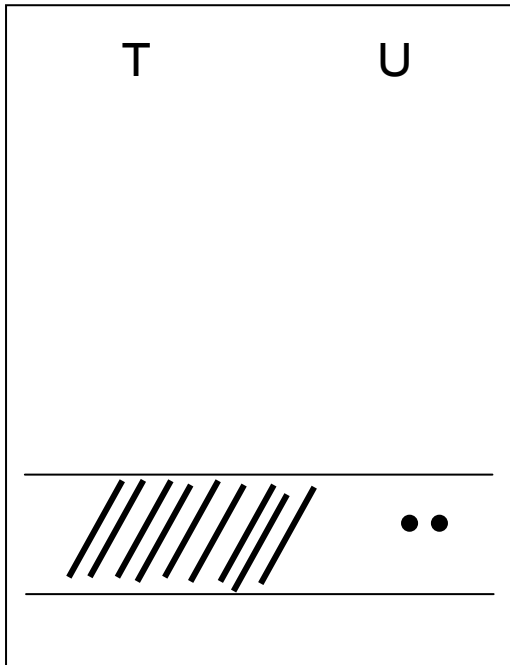
Step 2



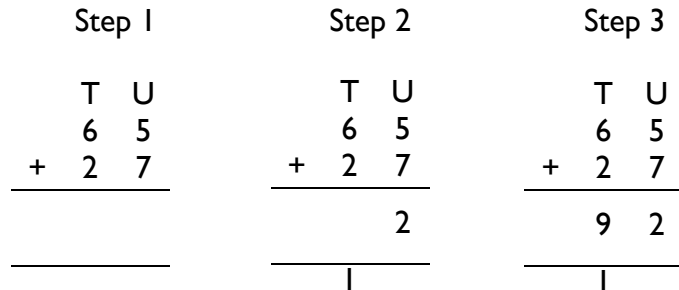
Children would exchange ten units/ones for a ten, placing the exchanged ten below the equals sign. Any remaining units/ones that cannot be exchanged for a ten move into the equals sign as they are the units part of the answer (as in the diagram in Step 2 above).

If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



Written method



Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially. This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method.

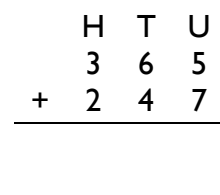
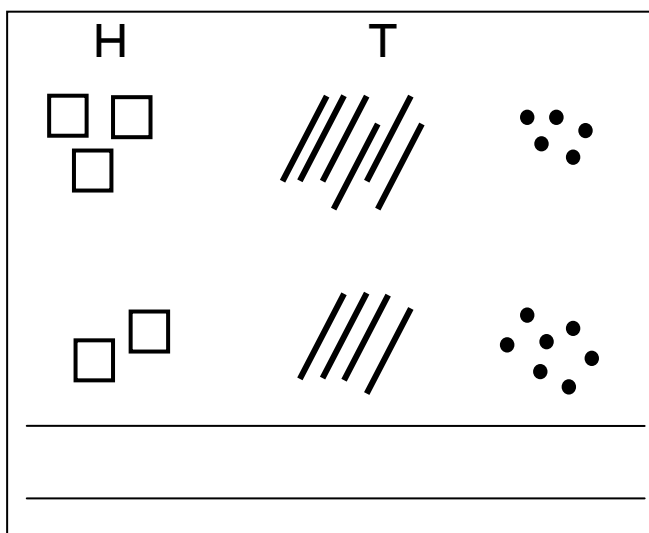
By the end of year 3, children should also extend this method for three digit numbers.

Y4

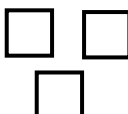

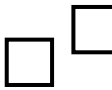

End of Year Objective:
Add numbers with up to 4 digits and decimals with one decimal place using the formal written method of columnar addition where appropriate.

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

Step 1

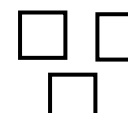
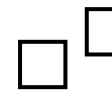



Step 2

H	T	U
		
		
		••
		/

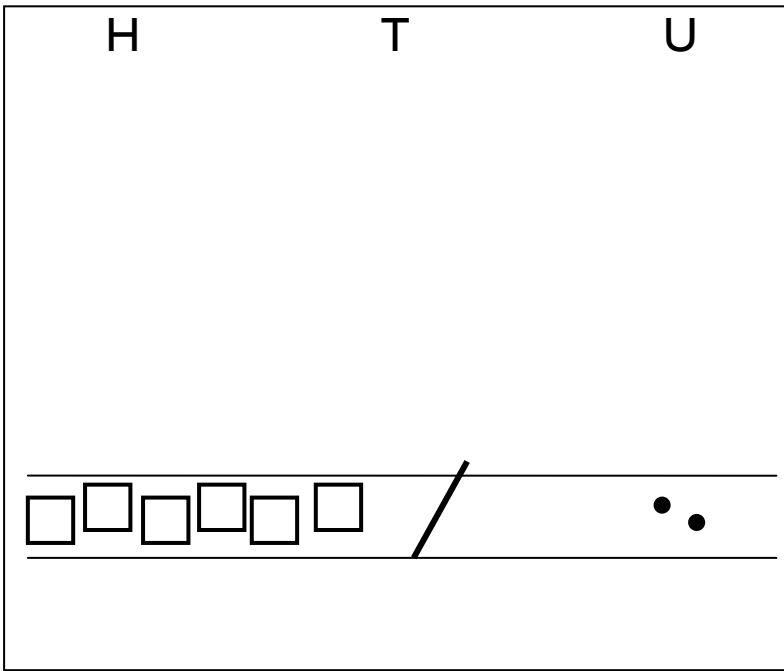
$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 3 \quad 6 \quad 5 \\
 + 2 \quad 4 \quad 7 \\
 \hline
 \quad 2 \\
 \hline
 1
 \end{array}$$

Step 3

H	T	U
		
		
		••
		

$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 3 \quad 6 \quad 5 \\
 + 2 \quad 4 \quad 7 \\
 \hline
 \quad 1 \quad 2 \\
 \hline
 1 \quad 1
 \end{array}$$

Step 4



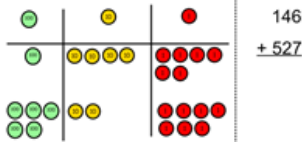
$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 3 \quad 6 \quad 5 \\
 + 2 \quad 4 \quad 7 \\
 \hline
 6 \quad 1 \quad 2 \\
 \hline
 1 \quad 1
 \end{array}$$

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

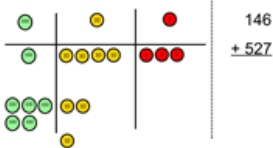
- several numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

Column method regrouping using place value counters

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

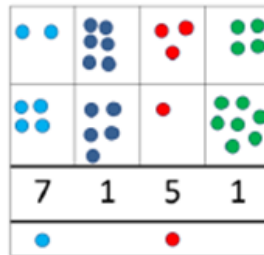


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Y5

End of Year Objective:

Add whole numbers with more than 4 digits *and* decimals with two decimal places, including formal written methods (columnar addition).

Children should continue to use the carrying method to solve calculations such as:

$$\begin{array}{r} 3364 \\ + 247 \\ \hline 3611 \\ \hline | | \end{array}$$

$$\begin{array}{r} 3121 \\ + 37 \\ \hline 148 \\ \hline 3306 \\ \hline | | \end{array}$$

$$\begin{array}{r} 3.56 \\ + 2.47 \\ \hline 6.03 \\ \hline | \end{array}$$

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Y6

End of Year Objective:

Add whole numbers and decimals using formal written methods (columnar addition).

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ + 3 \\ \hline 4681 \\ \hline | | 944 \\ \hline | | 2 | \end{array}$$

$$\begin{array}{r} 401.20 \\ + 26.85 \\ \hline 0.71 \\ \hline 428.76 \\ \hline | \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.