

Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5×3 is the same as 3×5
- associative i.e. $2 \times 3 \times 5$ is the same as $2 \times (3 \times 5)$

YR

Early Learning Goal:
Children solve problems, including doubling.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities to show how to double a number.</p> <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p style="text-align: center;">Double 4 is 8</p>	<p>Partition a number and then double each part before recombining it back together.</p>
Counting in multiples	<p>Count in multiples supported by concrete objects in equal groups.</p>	<p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Y1

End of Year Objective:















Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Y2

End of Year Objective:

Calculate mathematical statements for multiplication (*using repeated addition*) and write them using the multiplication (\times) and equals ($=$) signs.

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

<p>Repeated addition</p>	   <p>Use different objects to add equal groups.</p>	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>  <p>2 add 2 add 2 equals 6</p>  <p>5 + 5 + 5 = 15</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p>2 + 2 + 2 + 2 + 2 = 10</p>
<p>Arrays- showing commutative multiplication</p>	<p>Create arrays using counters/ cubes to show multiplication sentences.</p>  	<p>Draw arrays in different rotations to find commutative multiplication sentences.</p>  <p>$4 \times 2 = 8$</p>  <p>$2 \times 4 = 8$</p>  <p>$2 \times 4 = 8$</p>  <p>$4 \times 2 = 8$</p>  <p>Link arrays to area of rectangles.</p>	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>5 + 5 + 5 = 15</p> <p>3 + 3 + 3 + 3 + 3 = 15</p> <p>5 x 3 = 15</p> <p>3 x 5 = 15</p>

Y3

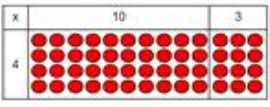
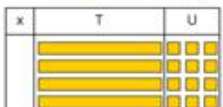

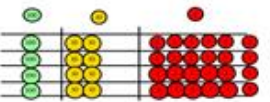
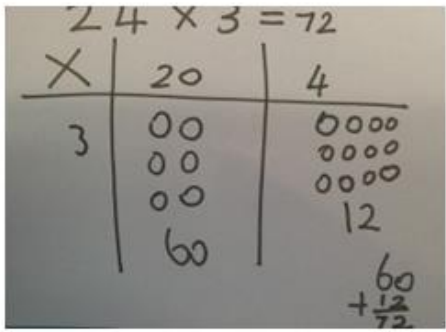
End of Year Objective:

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method. It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

<p>Show the link with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move on to using Base 10 to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126.</p>  <p>Calculations 4 x 126</p> <p>Add up each column, starting with the ones making any exchanges needed.</p>	<p>Children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1125 1355 1369 1429"> <tr> <td>X</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>210 + 35 = 245</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1141 1668 1380 1814"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table> <table border="1" data-bbox="1125 1848 1369 1960"> <tr> <td>X</td> <td>1000</td> <td>300</td> <td>40</td> <td>2</td> </tr> <tr> <td>10</td> <td>10000</td> <td>3000</td> <td>400</td> <td>20</td> </tr> <tr> <td>8</td> <td>8000</td> <td>2400</td> <td>320</td> <td>16</td> </tr> </table>	X	30	5	7	210	35		10	8	10	100	80	3	30	24	X	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16
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10	10000	3000	400	20																												
8	8000	2400	320	16																												

Y4

End of Year Objective:

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will move to year 4 using whichever method they were using as they transitioned from year 3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

79×8

x	70	9
8	560	72

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that $7 \times 8 = 56$ to know that $70 \times 8 = 560$.

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

346×8

x	300	40	6
8	2400	320	48

$$\begin{array}{r} 2400 \\ + 320 \\ + 48 \\ \hline 2768 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y5

End of Year Objective:

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

$$4346 \times 8$$

x	4 000	300	40	6
8	32 000	2400	320	48

$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g.

$$2693 \times 24$$

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12

$$\begin{array}{r} 40000 \\ + 8000 \\ + 12000 \\ + 2400 \\ + 1800 \\ + 360 \\ + 60 \\ + 12 \\ \hline 64632 \end{array}$$

The long list of numbers in the addition part can be used to check that all of the answers from the grid have been included, however, when children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they should be encouraged to do so. For example,

x	2000	600	90	3	
20	40000	12000	1800	60	= 53 860
4	8000	2400	360	12	= 10 772 +
					<hr/>
					64 632
					<hr/>

Adding across mentally, leads children to finding the separate answers to:

$$2\,693 \times 20$$

$$2\,693 \times 4$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

During Year 5, the transition from the grid method into the formal vertical method for multiplication should take place. The traditional vertical compact method of written multiplication is a highly efficient way to calculate, but it has a very condensed form and needs to be introduced carefully. It is most effective to begin with the grid method, moving to an expanded vertical layout, before introducing the compact form. This allows children to see, and understand, how the processes relate to each other and where the individual multiplication answers come from e.g.

$$368 \times 6$$

x	300	60	8	
6	1 800	360	48	

$$\begin{array}{r}
 1800 \\
 + 360 \\
 + 48 \\
 \hline
 2208
 \end{array}$$

Th	H	T	U	
3	6	8		
x			6	
		4	8	(8 × 6)
	3	6	0	(60 × 6)
+ 1	8	0	0	(300 × 6)
	2	2	0	8

Th	H	T	U	
3	6	8		
x			6	
		4	8	(8 × 6)
	3	6	0	(60 × 6)
+ 1	8	0	0	(300 × 6)
	2	2	0	8

becomes

Th	H	T	U
3	6	8	
x			6
		2	2
	0	0	8

The place value columns are labelled to ensure children understand the size of the partitioned digits in the original number(s) and in the answer.

It is vital that the teacher models the correct language when explaining the process of the compact method.

The example shown should be explained as:

“Starting with the least significant digit... 8 multiplied by 6 is 48, put 8 in the units and carry 4 tens (40). 6 tens multiplied by 6 are 36 tens. Add the 4 tens carried over to give 40 tens (which is the same as 4 hundreds and 0 tens). Put 0 in the tens place of the answer and carry 4 hundreds. 3 hundreds multiplied by 6 are 18 hundreds. Add the 4 hundreds carried over to give 22 hundreds (which is the same as 2 thousands and 2 hundreds). Write 2 in the hundreds place of the answer and 2 in the thousands place of the answer.”

Children should recognise that the answer is close to an estimated answer of $400 \times 6 = 2\,400$

Long multiplication could also be introduced by comparing the grid method with the compact vertical method. Mentally totalling each row of answers is an important step in children making the link between the grid method and the compact method.

x	600	90	3	
20	12000	1800	60	= 13 860
4	2400	360	12	= 2 772 +
				16 632

4.92×3

$$\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline 0 . 0 6 \quad (0.02 \times 3) \\ 2 . 7 \quad (0.9 \times 3) \\ + 1 2 \quad (4 \times 3) \\ \hline \underline{1 4 . 7 6} \end{array}$$

becomes

$$\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline \underline{1 4 . 7 6} \\ \quad \quad \quad 2 \end{array}$$

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.