

Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

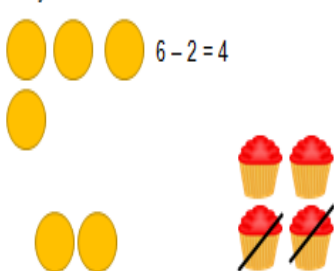
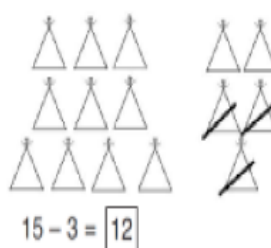


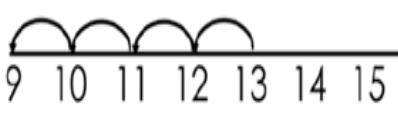
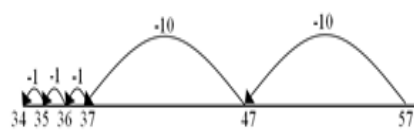
- the inverse of addition
- not commutative i.e. $5 - 3$ is not the same as $3 - 5$
- not associative i.e. $10 - 3 - 2$ is not the same as $10 - (3 - 2)$

YR


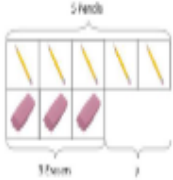
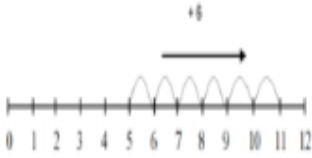
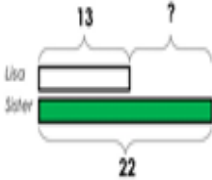

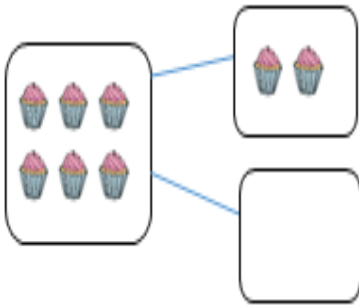

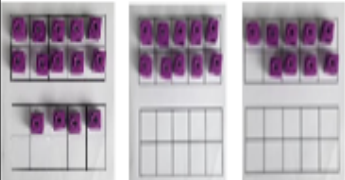
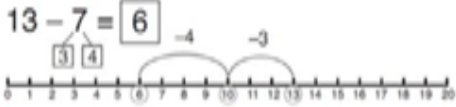
Early Learning Goal:

Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 2 = 4$</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p>	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>
<p>Counting back</p>	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>

End of Year Objective:
Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations). Understand subtraction as finding the difference.

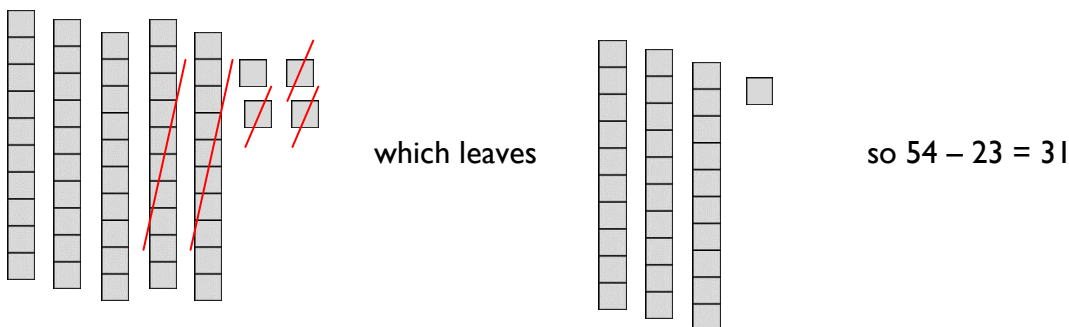
<p>Find the difference</p>	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference</p>	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Draw bars to find the difference between 2 numbers.</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p> 	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
<p>Part Part Whole Model</p>	<p>Link to addition-use the part whole model to help explain the inverse between addition and subtraction.</p>  <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
<p>Make 10</p>	<p>$14 - 9 =$</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.</p>	<p>$13 - 7 = 6$</p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>$16 - 8 =$</p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p>

Y2

End of Year Objective:

Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers.

Children will begin to use the Base 10 equipment to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate $54 - 23$, children would count out 54 using the Base 10 equipment (5 tens and 4 units). They need to consider whether there are enough units/ones to remove 3, in this case there are, so they would remove 3 units and then two tens, counting up the answer of 3 tens and 1 unit to give 31.



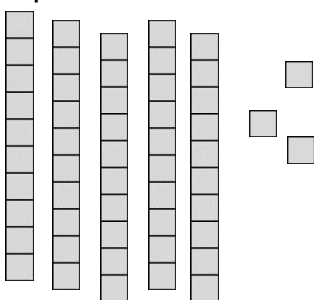
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks), e.g. to calculate $39 - 17$ children would draw 39 as 3 tens (lines) and 4 units (dots) and would cross out 7 units and then one ten, counting up the answer of 2 tens and 2 units to give 22.



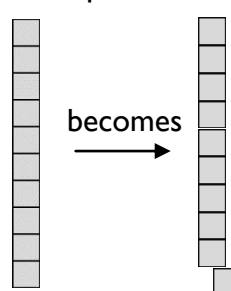
Circling the tens and units that remain will help children to identify how many remain.

When the amount of units to be subtracted is greater than the units in the original number, an exchange method is required. This relies on children's understanding of ten units being an equivalent amount to one ten. To calculate $53 - 26$, by using practical equipment, they would count out 53 using the tens and units, as in Step 1. They need to consider whether there are enough units/ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1

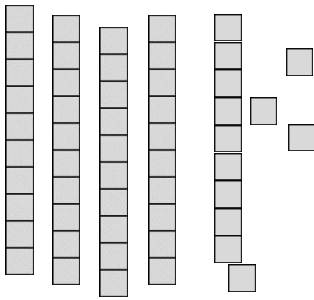


Step 2

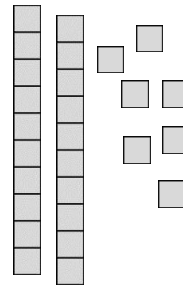


The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.

Step 3

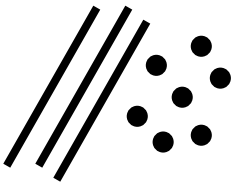


Step 4

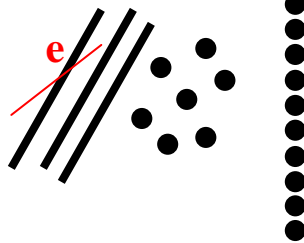


When recording their own drawings, when calculating $37 - 19$, children would cross out a ten and exchange for ten units. The exchanged ten is denoted with an **e** so children recognise this has not been subtracted. Drawing the units in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the units that were already in place.

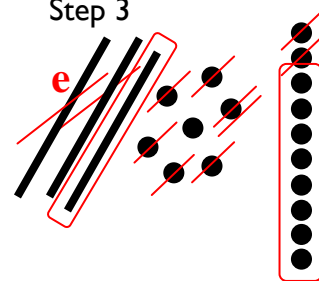
Step 1



Step 2



Step 3



Circling the tens and units that remain will help children to identify how many remain.

End of Year Objective:
Subtract numbers with up to three digits, using formal written method of columnar subtraction.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using Base 10 equipment from year 2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the units and tens as appropriate (as with the more informal method in year 2).

Step 1

$$\begin{array}{r} 80 \\ - 50 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 7 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} 80 \\ - 50 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 7 \\ \hline \end{array}$$

Step 3

$$\begin{array}{r} 80 \\ - 50 \\ \hline 30 \end{array}$$


$$\begin{array}{r} 9 \\ - 7 \\ \hline 2 \end{array}$$

Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.

This will be recorded by the children as:

$$\begin{array}{r} 80 \rightarrow 9 \\ - 50 \rightarrow 7 \\ \hline 30 \rightarrow 2 \end{array} = 32$$


Children can also use jottings of the Base 10 materials (as in year 2) to support with their calculation, as in the example below.



$$\begin{array}{r}
 80 \rightarrow 9 \\
 - 50 \rightarrow 7 \\
 \hline
 30 \rightarrow 2 = 32
 \end{array}$$


From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough units/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same ($71 = 70 + 1 = 60 + 11$).

Step 1




$$\begin{array}{r}
 70 \\
 - 40 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 1 \\
 - 6 \\
 \hline
 \end{array}$$

Step 2




$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 \end{array}$$

Step 3



$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 \end{array}$$

Step 4



$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 20 \\
 \end{array}
 \quad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 5 \\
 \end{array}$$

This will be recorded by the children as:

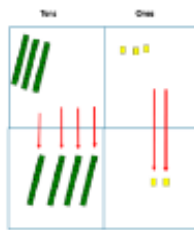
$$\begin{array}{r}
 60 \\
 70 \rightarrow 11 \\
 - 40 \rightarrow 6 \\
 \hline
 20 \rightarrow 5 = 25
 \end{array}$$

By the end of year 3, children should also extend this method for three digit numbers.

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

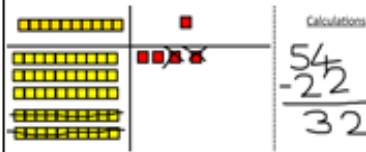
concrete	pictorial	Abstract
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Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.



Calculations

$$\begin{array}{r} 176 - 64 = \\ 176 \\ - 64 \\ \hline 112 \end{array}$$

$$47 - 24 = 23$$

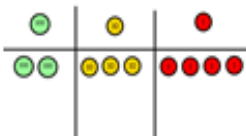
$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

This will lead to a clear written column subtraction.

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

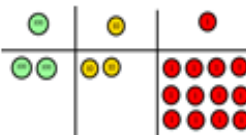
Make the larger number with the place value counters



Calculations

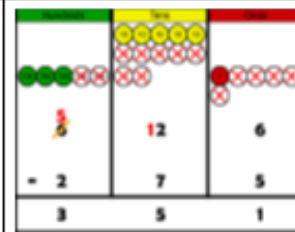
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

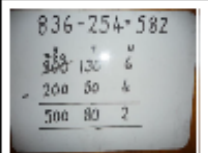


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

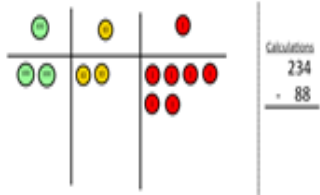


Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

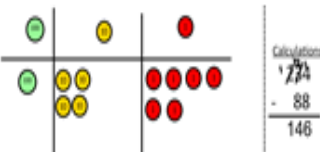
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

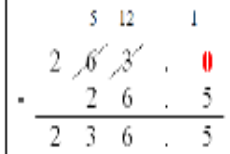


Now I can take away eight tens and complete my subtraction



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

This will lead to an understanding of subtracting any number including decimals.



Yr4

Ensure children can partition numbers to subtract using base 10 and place value counters before progressing

End of Year Objective:

Subtract numbers with up to 4 digits *and* decimals with one decimal place using the formal written method of columnar subtraction where appropriate.

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

Step 1

$$\begin{array}{r} 700 \rightarrow 50 \rightarrow 4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \hline \end{array}$$

Step 2 (exchanging from tens to units)

$$\begin{array}{r} 700 \rightarrow \overset{40}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \hline \end{array}$$

Step 3 (exchanging from hundreds to tens)

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \hline \end{array}$$

Step 4

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

This would be recorded by the children as:

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

When children are ready, this leads on to the compact method of decomposition:

$$\begin{array}{r} 4 \overset{6}{\cancel{7}} \overset{14}{\cancel{5}} \overset{1}{4} \\ - 3 \quad 2 \quad 8 \quad 6 \\ \hline 1 \quad 4 \quad 6 \quad 8 \end{array}$$

By the end of year 4, children should be using the written method confidently and with understanding. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

Y5

End of Year Objective:

Subtract whole numbers with more than 4 digits *and decimals with two decimal places, including formal written methods (columnar subtraction).*

Children should continue to use the decomposition method to solve calculations such as:

$$\begin{array}{r} \overset{6}{7} \overset{6}{10} \overset{6}{7} \overset{6}{12} \\ - \quad 3 \quad 2 \quad 2 \quad 6 \\ \hline 3 \quad 8 \quad 4 \quad 6 \end{array}$$

$$\begin{array}{r} \overset{2}{3} \overset{13}{4} \overset{12}{12} \\ - \quad 1 \quad . \quad 7 \quad 6 \\ \hline 1 \quad . \quad 6 \quad 6 \end{array}$$

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Y6

End of Year Objective:

Subtract whole numbers and decimals using formal written methods (columnar subtraction).

Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

$$\begin{array}{r} \overset{5}{6} \overset{13}{4} \overset{13}{13} \overset{2}{2} \\ - \quad 4 \quad 6 \quad 8 \quad 1 \\ \hline 1 \quad 7 \quad 5 \quad 1 \end{array}$$

$$\begin{array}{r} \overset{3}{4} \overset{6}{11} \overset{6}{7} \overset{11}{2} \overset{10}{0} \\ - \quad 3 \quad 4 \quad . \quad 7 \quad 1 \\ \hline 3 \quad 8 \quad 2 \quad . \quad 4 \quad 9 \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.