Brilliant Maths Revision

GREEN PACK

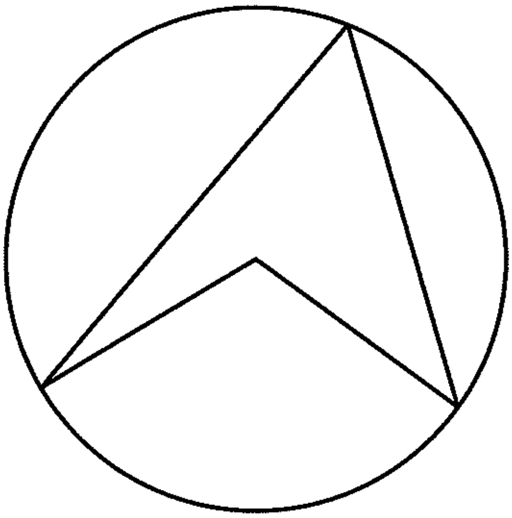
Grade 8/9

Name:

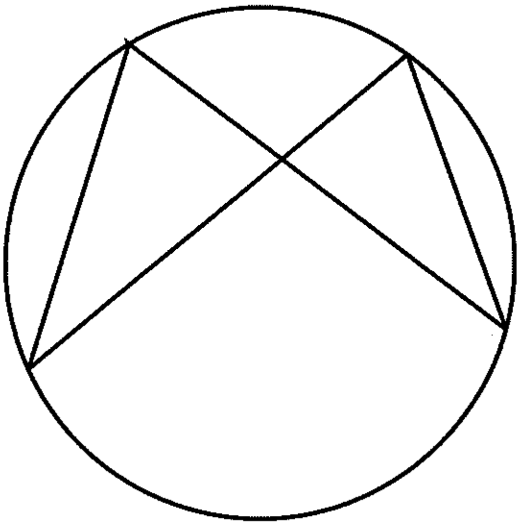
If you complete this pack, you can trade it in for the next grade up – ask your maths teacher

Proof of Circle Theorems

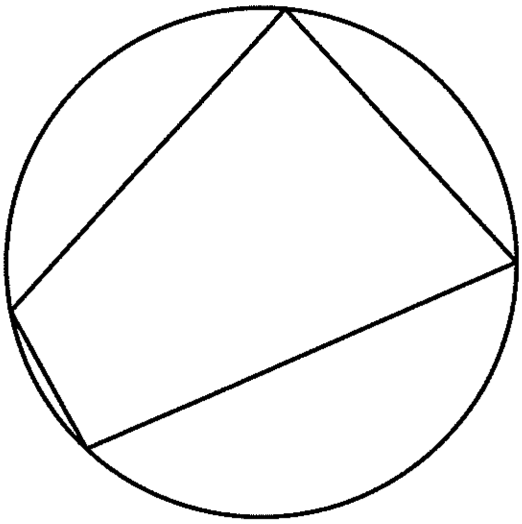
1) Prove that the angle subtended at the centre of a circle is twice the angle at the circumference.



2) Prove that angles in the same segment are equal.



3) Prove that opposite angles of a cyclic quadrilateral add up to 180°.



Factorising Hard Quadratics

1) Factorise the following:

a) $2x^2 + 7x + 3$

b) $3x^2 + 5x - 2$

c) $6x^2 - 11x + 3$

d) $8x^2 + 10x + 3$

e) $6x^2 - 7x - 20$

f) $4x^2 - 4x - 15$

2) Solve the following:

a) $5x^2 + 9x - 2 = 0$

b) $6x^2 + 5x - 6 = 0$

c) $12x^2 + 25x + 7 = 0$

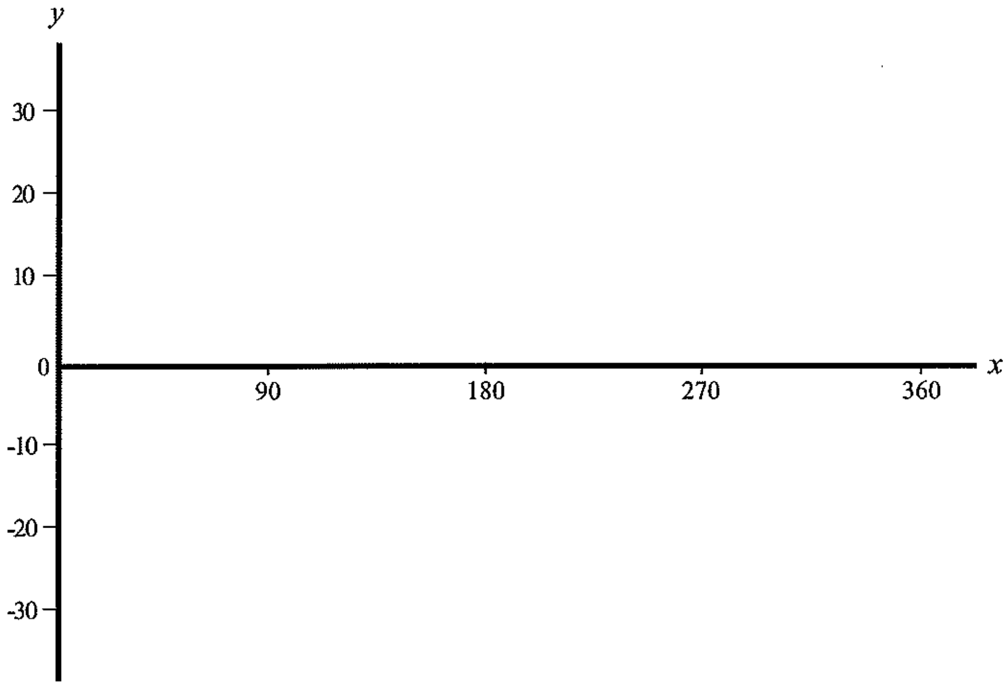
d) $8x^2 - 14x - 15 = 0$

e) $10x^2 - 43x - 30 = 5$

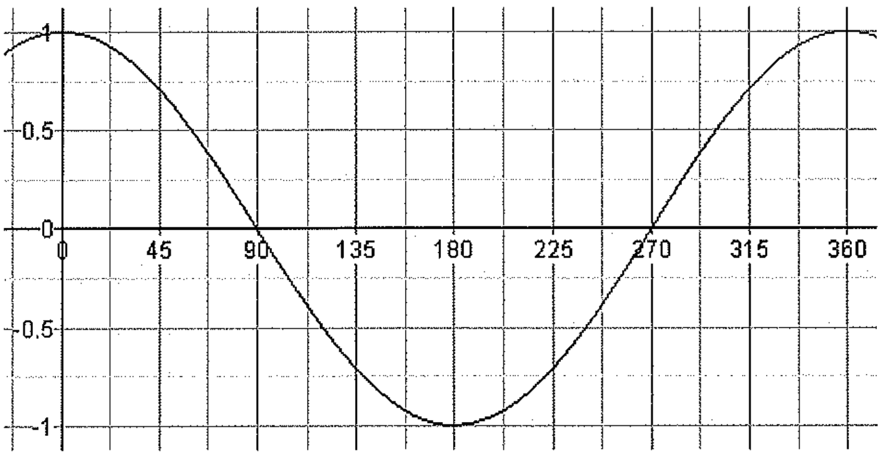
f) $4x^2 - 8x + 2 = 7$

Trigonometric Graphs

1) On the axes below, draw a sketch-graph to show $y = \tan x$



2) Here is the graph of the curve $y = \cos x$ for $0 \leqslant x \leqslant 360°$.

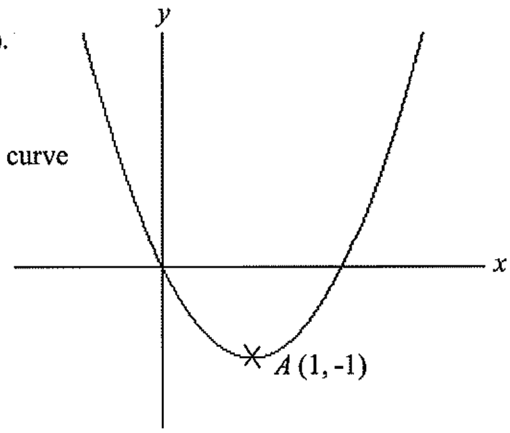


a) Use the graph to solve $\cos x = 0.75$ for $0 \leqslant x \leqslant 360°$

b) Use the graph to solve $\cos x = -0.75$ for $0 \leqslant x \leqslant 360°$

Transformation of Functions

1) This is a sketch of the curve with equation $y = f(x)$.
It passes through the origin O.
The only vertex of the curve is at A (1, -1)

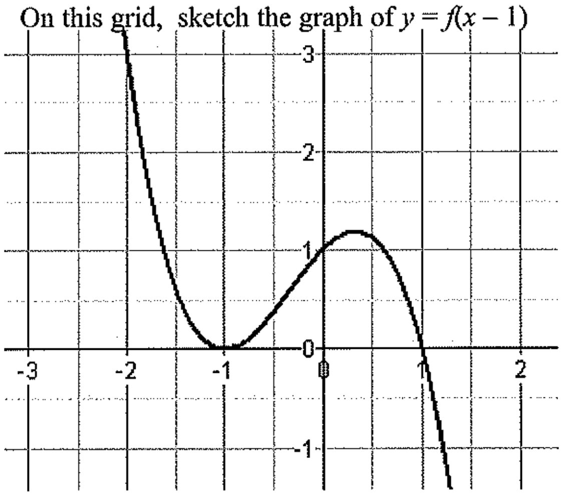


a) Write down the coordinates of the vertex of the curve with equation
   (i) $y = f(x - 3)$
   (ii) $y = f(x) - 5$
   (iii) $y = -f(x)$

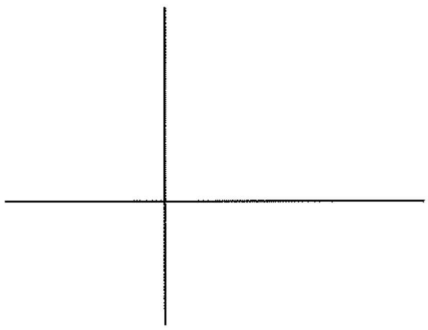
b) The curve $y = x^2$ has been translated to give the curve $y = f(x)$.
   Find $f(x)$ in terms of x.

2) The graph of $y = f(x)$ is shown on the grids.

On this grid, sketch the graph of $y = f(x - 1)$

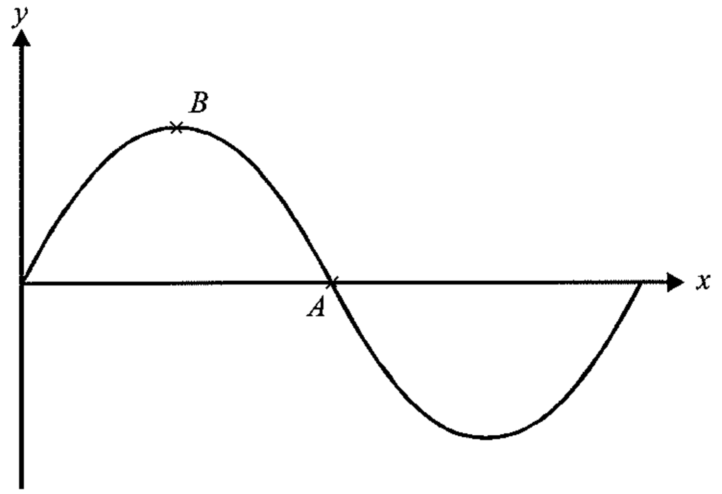


3) Sketch the graph of $y = (x - 2)^2 + 3$
State the coordinates of the vertex.



Transformation of Functions

1) The diagram below shows the graph of $y = 2\sin x$, for values of x between 0 and 360°.



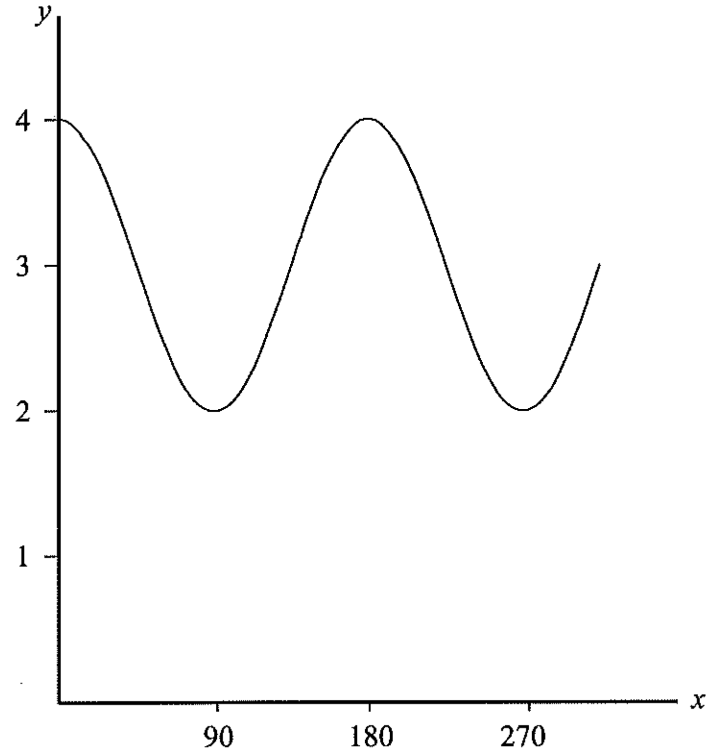
The curve cuts the x axis at the point A.
The graph has a maximum at the point B.

a) (i) Write down the coordinates of A.
   (ii) Write down the coordinates of B.

b) On the same diagram, sketch the graph of $y = 2\sin x + 1$ for values of x between 0° and 360°.

2) The diagram below shows the graph of $y = \cos \boldsymbol{a}x + \boldsymbol{b}$, for values of x between 0° and 300°.

Work out the values of ***a*** and ***b***.



Direct and Inverse Proportion

1) x is directly proportional to y.
When $x = 21$, then $y = 3$.

a) Express x in terms of y.

b) Find the value of x when y is equal to 10.

2) a is inversely proportional to b.
When $a = 12$, then $b = 4$.

a) Find a formula for a in terms of b.

b) Find the value of a when b is equal to 8.

c) Find the value of b when a is equal to 4.

3) The variables u and v are in inverse proportion to one another.
When $u = 3$, then $v = 8$.

Find the value of u when $v = 12$.

4) p is directly proportional to the square of q.
$p = 75$ when $q = 5$

a) Express p in terms of q.

b) Work out the value of p when $q = 7$.

c) Work out the positive value of q when $p = 27$.

5) y is directly proportional to x^2.
When $x = 3$, then $y = 36$.

a) Express y in terms of x.

z is inversely proportional to x.
When $x = 4$, $z = 2$.

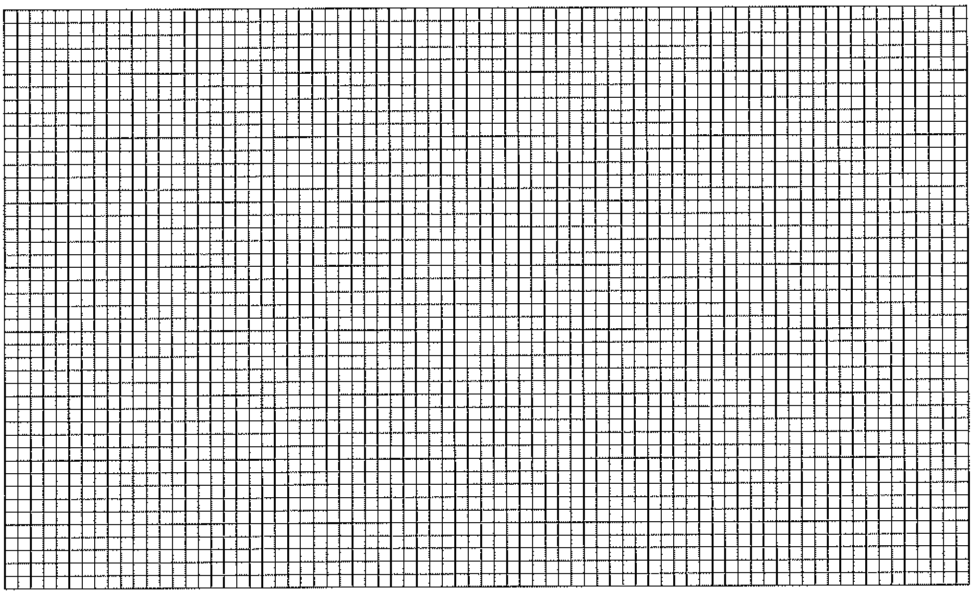
b) Show that $z = c\,y^n$, where c and n are numbers and $c > 0$.
You must find the values of c and n.

Histograms

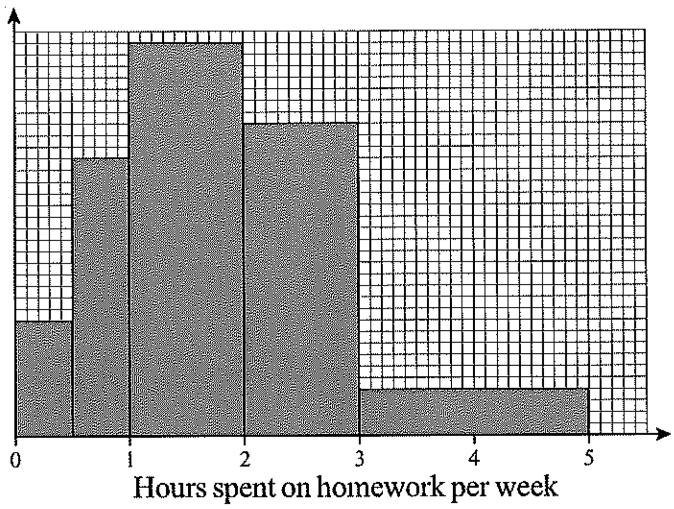
1) The table gives information about the heights, in centimetres, of some 18 year old students.

Use the table to draw a histogram.

Height (h cm)	Frequency
$135 < h \leqslant 145$	12
$145 < h \leqslant 165$	46
$165 < h \leqslant 180$	45
$180 < h \leqslant 190$	25
$190 < h \leqslant 195$	4



2) The histogram shows the amount of time, in hours, that students spend on their homework per week.

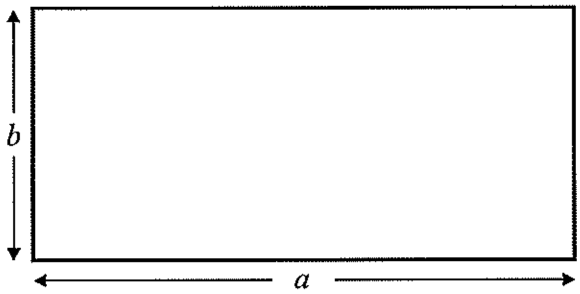


Use the histogram to complete the table.

Time (t hours)	Frequency
$0 < t \leqslant ½$	
$½ < t \leqslant 1$	
$1 < t \leqslant 2$	
$2 < t \leqslant 3$	27
$3 < t \leqslant 5$	

Upper and Lower Bounds

1)



The length of the rectangle, a,
is 45 cm correct to the nearest cm.

The width of the rectangle, b,
is 26 cm correct to the nearest cm.

Calculate the upper bound for the area of the rectangle.
Write down all the figures on your calculator display.

2) A field is in the shape of a rectangle.
The width of the field is 26 metres, measured to the nearest metre.

a) Work out the upper bound of the width of the field.

The length of the field is 135 metres, measured to the nearest 5 metres.

b) Work out the upper bound for the perimeter of the field.

3) A ball is thrown vertically upwards with a speed V metres per second.

The height, H metres, to which it rises is given by

$$H = \frac{V^2}{2g}$$

where g m/s^2 is the acceleration due to gravity.

$V = 24.4$ correct to 3 significant figures.
$g = 9.8$ correct to 2 significant figures.

(i) Write down the lower bound of g.

(ii) Calculate the upper bound of H.
Give your answer correct to 3 significant figures.

4) $v = \sqrt{\frac{a}{b}}$

$a = 6.43$ correct to 2 decimal places.
$b = 5.514$ correct to 3 decimal places.

By considering bounds, work out the value of v to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

Upper and Lower Bounds

1) $A = 11.3$ correct to 1 decimal place
$B = 300$ correct to 1 significant figure
$C = 9$ correct to the nearest integer

a) Calculate the upper bound for $A + B$.

b) Calculate the lower bound for $B \div C$.

c) Calculate the least possible value of AC.

d) Calculate the greatest possible value of $\dfrac{A+B}{B+C}$

2) An estimate of the acceleration due to gravity can be found using the formula:

$$g = \frac{2L}{T^2 \sin x}$$

Using
$T = 1.2$ correct to 1 decimal place
$L = 4.50$ correct to 2 decimal places
$x = 40$ correct to the nearest integer

a) Calculate the lower bound for the value of g.
Give your answer correct to 3 decimal places.

b) Calculate the upper bound for the value of g.
Give your answer correct to 3 decimal places.

3) The diagram shows a triangle ABC.

$AB = 73$mm correct to 2 significant figures.
$BC = 80$mm correct to 1 significant figure.

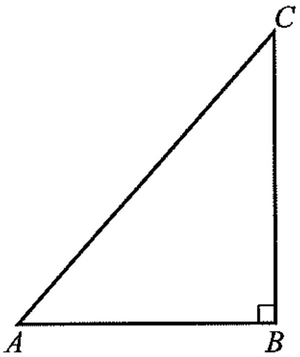


Diagram **NOT** accurately drawn

a) Write the upper and lower bounds of both AB and BC.

AB_{upper} = ... BC_{upper} = ...
AB_{lower} = ... BC_{lower} = ...

b) Calculate the upper bound for the area of the triangle ABC.

........................mm²

Angle $CAB = x°$
c) Calculate the lower bound for the value of $\tan x°$.

Surds

1) Simplify the following:

a) $\sqrt{7} \times \sqrt{7}$

b) $\sqrt{3} \times \sqrt{3}$

c) $\sqrt{20}$

d) $\sqrt{24}$

e) $\sqrt{72}$

f) $\sqrt{200}$

g) $\sqrt{\frac{2}{25}}$

2) Simplify the following:

a) $\sqrt{2} \times \sqrt{18}$

b) $\sqrt{8} \times \sqrt{32}$

c) $\sqrt{99} \times \sqrt{22}$

d) $\sqrt{45} \times \sqrt{20}$

e) $\sqrt{18} \times \sqrt{128}$

f) $\sqrt{28} \times \sqrt{175}$

3) Expand and simplify where possible:

a) $\sqrt{3}(3 - \sqrt{3})$

b) $\sqrt{2}(6 + 2\sqrt{2})$

c) $\sqrt{7}(2 + 3\sqrt{7})$

d) $\sqrt{2}(\sqrt{32} - \sqrt{8})$

4) Expand and simplify where possible:

a) $(1 + \sqrt{2})(1 - \sqrt{2})$

b) $(3 + \sqrt{5})(2 - \sqrt{5})$

c) $(\sqrt{3} + 2)(\sqrt{3} + 4)$

d) $(\sqrt{5} - 3)(\sqrt{5} + 1)$

e) $(2 + \sqrt{7})(2 - \sqrt{7})$

f) $(\sqrt{6} - 3)^2$

5) Work out the following, giving your answer in its simplest form:

a) $\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{22}}$

b) $\frac{(4 - \sqrt{5})(4 + \sqrt{5})}{\sqrt{11}}$

c) $\frac{(3 - \sqrt{2})(3 + \sqrt{2})}{\sqrt{14}}$

d) $\frac{(\sqrt{3} + 1)^2}{\sqrt{3}}$

e) $\frac{(\sqrt{5} + 3)^2}{\sqrt{20}}$

f) $\frac{(5 - \sqrt{5})(2 + 2\sqrt{5})}{\sqrt{20}}$

Surds

1) $\sqrt{5} = 5^k$

a) Write down the value of k.

b) Expand and simplify $(2 + \sqrt{5})(1 + \sqrt{5})$

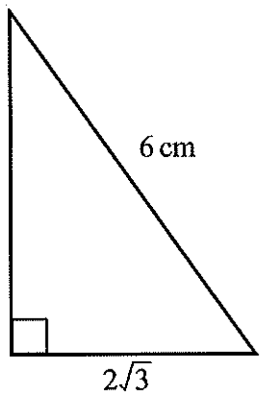
Give your answer in the form $a + b\sqrt{c}$

where a, b and c are integers.

2) The diagram shows a right-angled triangle with lengths of sides as indicated.

The area of the triangle is A cm^2

Show that $A = k\sqrt{2}$ giving the value of k.



3) Given that

$$\frac{8 - \sqrt{18}}{\sqrt{2}} = a + b\sqrt{2}$$, where a and b are integers,

find the value of a and the value of b.

4) Work out $(2 + \sqrt{3})(2 - \sqrt{3})$

Give your answer in its simplest form.

Surds

1) Rationalise the denominator, simplifying where possible:

a) $\frac{3}{\sqrt{2}}$

b) $\frac{2}{\sqrt{2}}$

c) $\frac{3\sqrt{2}}{\sqrt{7}}$

d) $\frac{\sqrt{5}}{\sqrt{10}}$

e) $\frac{1}{4\sqrt{8}}$

f) $\frac{\sqrt{15}}{\sqrt{3}}$

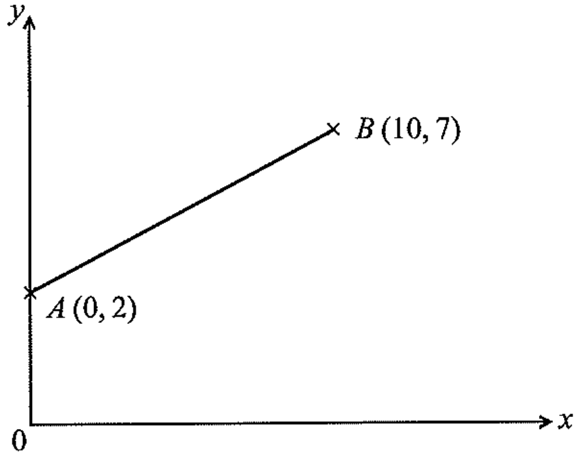
g) $\frac{1}{\sqrt{27}}$

2) Rationalise the denominator of $\frac{1}{\sqrt{3}}$

3) Rationalise the denominator of $\frac{1}{8\sqrt{8}}$ giving the answer in the form $\frac{\sqrt{2}}{p}$

Perpendicular Lines

1)



A is the point (0, 2)
B is the point (10, 7)

a) Write down the equation of the straight line which passes through points A and B.

b) Find the equation of the line perpendicular to AB passing through B.

2) A straight line has equation $y = 2x - 5$
The point P lies on the straight line.
The y coordinate of P is -6

a) Find the x coordinate of P.

A straight line L is parallel to $y = 2x - 5$ and passes through the point (3, 2).

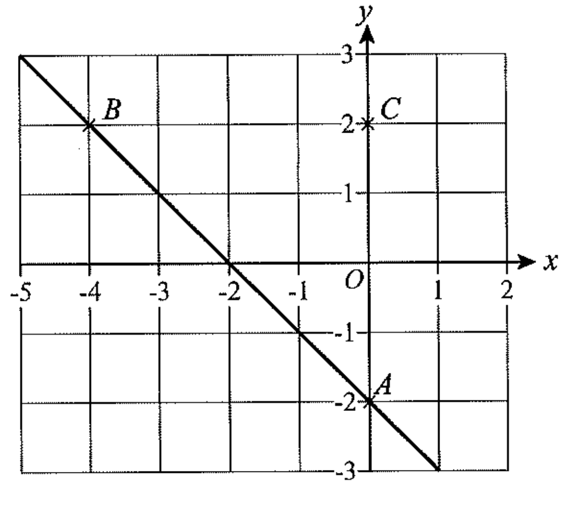
b) Find the equation of line L.

c) Find the equation of the line that is perpendicular to line L and passes through point (3, 2).

3) In the diagram A is the point (0, -2)
B is the point (-4, 2)
C is the point (0, 2)

a) Find the equation of the line that passes through C and is parallel to AB.

b) Find the equation of the line that passes through C and is perpendicular to AB.



Completing the Square

1) Show that if $y = x^2 + 8x - 3$
then $y \geq -19$ for all values of x.

2) Show that if $y = x^2 - 10x + 30$
then $y \geq 5$ for all values of x.

3) The expression $x^2 + 4x + 10$ can be written in the form
$(x + p)^2 + q$ for all values of x.
Find the values of p and q.

4) Given that $x^2 - 6x + 17 = (x - p)^2 + q$ for all values of x,
find the value of p and the value of q.

5) For all values of x,

$$x^2 + 6x = (x + p)^2 + q$$

a) Find the values of p and q.

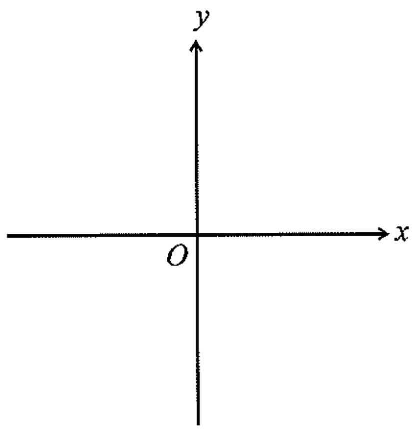
b) Find the minimum value of $x^2 + 6x$.

6) For all values of x,

$$x^2 - 8x - 5 = (x - p)^2 + q$$

a) Find the value of p and the value of q.

b) On the axes, sketch the graph of $y = x^2 - 8x - 5$.



c) Find the coordinates of the minimum point on the graph of $y = x^2 - 8x - 5$.

7) The expression $10x - x^2$ can be written in the form $p - (x - q)^2$ for all values of x.

a) Find the values of p and q.

b) The expression $10x - x^2$ has a maximum value.

(i) Find the maximum value of $10x - x^2$.

(ii) State the value of x for which this maximum value occurs.

Algebraic Fractions

1) Simplify fully

a) $\frac{9x^2}{21x^3}$

b) $\frac{10xy^3}{5y^2}$

c) $\frac{18a^3b^2}{2ab^2}$

d) $\frac{4x^2+12x}{10x}$

e) $\frac{2a^2b-14a^2b^3}{6a^3b^3}$

f) $\frac{5x^2y+5xy^2}{10x^2y^2}$

2) Simplify fully

a) $\frac{x^2+x}{x^2+6x+5}$

b) $\frac{x^2-6x+8}{2x^2-8x}$

c) $\frac{x^2+7x+10}{x^2+5x}$

3) a) Factorise $4x^2-12x+9$

b) Simplify $\frac{6x^2-7x-3}{4x^2-12x+9}$

Algebraic Fractions

1) Write as single fractions in their simplest form

a) $\frac{3}{x} + \frac{3}{2x}$

b) $\frac{5}{3x} - \frac{3}{4x}$

c) $\frac{x+2}{5} + \frac{x-1}{2}$

d) $\frac{3}{x+2} - \frac{5}{2x+1}$

2) a) Factorise $2x^2 + 7x + 6$

b) Write as a single fraction in its simplest form $\frac{3}{x+2} + \frac{4x}{2x^2 + 7x + 6}$

3) Solve

a) $\frac{1}{x} + \frac{1}{3x} = 2$

b) $\frac{1}{x-2} + \frac{3}{x+6} = \frac{1}{2}$

c) $\frac{1}{x-5} + \frac{6}{x} = 2$

d) $\frac{7}{x+2} + \frac{1}{x-1} = 4$

e) $\frac{3}{x+2} + \frac{1}{x-2} = \frac{7}{x^2 - 4}$

f) $\frac{x}{2x-1} + \frac{2}{x+2} = 1$

Simultaneous Equations with a Quadratic

1) Solve these simultaneous equations.

$y = x$

$y = x^2 - 6$

2) Solve these simultaneous equations.

$y = x^2 - 4$

$y = 3x$

3) Solve these simultaneous equations.

$y = x^2 - x - 13$

$y = x + 2$

4) Solve these simultaneous equations.

$y = x^2 - 35$

$x - y = 5$

5) Solve these simultaneous equations.

$x^2 + y^2 = 26$

$y + 6 = x$

6) Sarah said that the line $y = 7$ cuts the curve $x^2 + y^2 = 25$ at two points.

a) By eliminating y show that Sarah is **not** correct.

b) By eliminating y, find the solutions to the simultaneous equations

$x^2 + y^2 = 25$

$y = 3x - 9$

Solving Quadratic Inequalities

1) Solve the inequality $x^2 + x - 12 \leqslant 0$

2) Solve the inequality $x^2 - 5x - 6 > 0$

3) Solve the inequality $x^2 - 9 \geqslant 0$

4) Solve the inequality $2x^2 - 13x + 15 < 0$

5) Solve the inequality $6x^2 + 13x - 5 > 0$

Inverse Functions

1) Find $f^{-1}(x)$ if $f(x) = \frac{x}{4} + 3$

2) a) Find $f^{-1}(x)$ where $f(x) = 2x - 3$

b) Find $f^{-1}(19)$

3) a) Find $f^{-1}(x)$ where $f(x) = x^3 - 1$

b) Find $f^{-1}(26)$

4) Find $f^{-1}(x)$ where $f(x) = \frac{4x - 1}{x}$

5) Find $f^{-1}(x)$ where $f(x) = \frac{2x}{x + 5}$

Composite Functions

1) For all values of x,

$f(x) = x^2 - 2$, $g(x) = x + 6$

a) Find f(5)

b) Find f(-1)

c) Find g(3)

d) Find g(-5)

2) For all values of x,

$f(x) = x^2 - 2$, $g(x) = x + 6$

a) Find fg(3)

b) Find gf(3)

c) Find gf(0)

3) For all values of x,

$f(x) = x^2 + 3x$, $g(x) = x + 5$

a) Find $fg(x)$

b) Find $gf(x)$

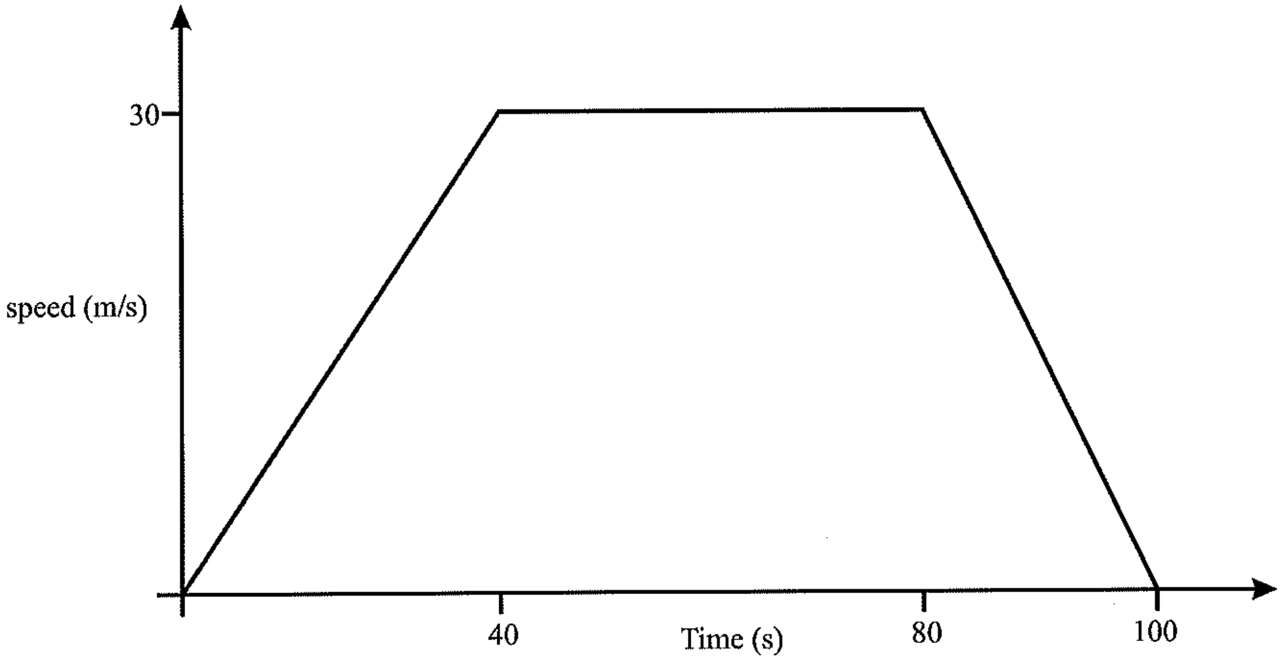
4) For all values of x,

$f(x) = x + 5$, $g(x) = x^2 - 2$

Solve $fg(x) = gf(x)$

Velocity-Time Graphs

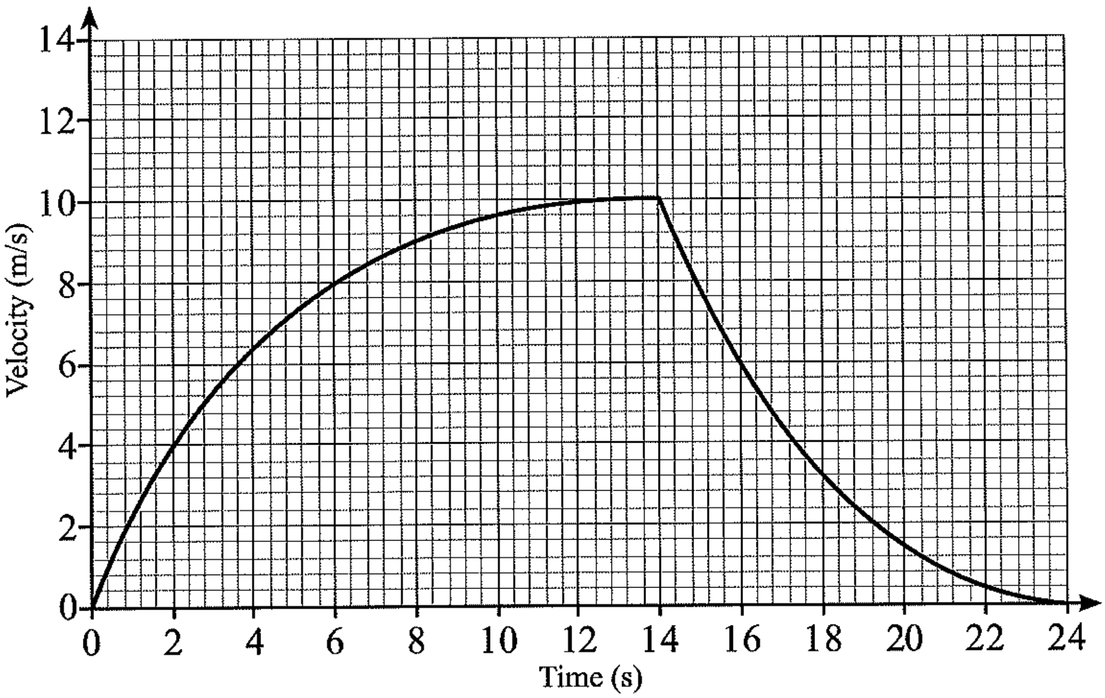
1) The graph shows the speed of a coach between two bus stations.



a) What was the acceleration of the coach in m/s^2 for the first 40 seconds?

b) What is the distance, in metres, between the two stations?

2) The velocity-time graph for a car is shown.



a) Estimate the acceleration of the car at 6 seconds.

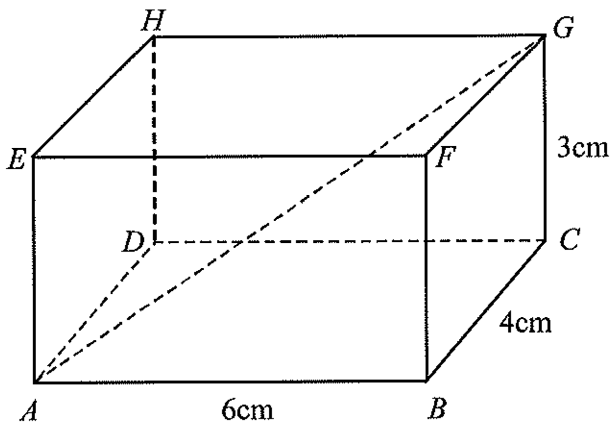
b) Find an estimate for how far the car has travelled in the first 14 seconds.
Show all your working.

Pythagoras in 3D

1) The diagram shows a box in the shape of a cuboid.
$AB = 6\text{cm}$, $BC = 4\text{cm}$, $CG = 3\text{cm}$

A string runs diagonally across the box from A to G.

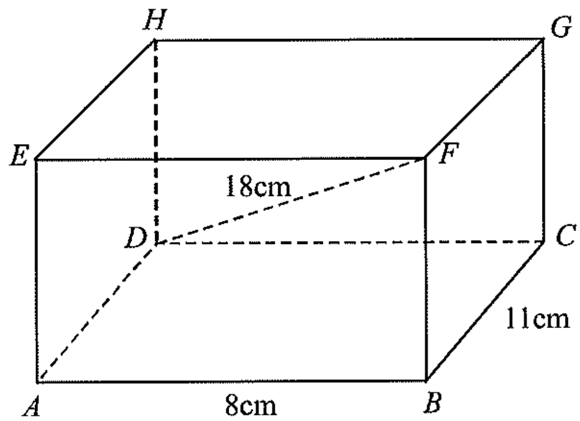
Calculate the length of the string AG.
Give your answer correct to 3 significant figures.



2) The diagram shows a box in the shape of a cuboid.
$AB = 8\text{cm}$, $BC = 11\text{cm}$

A string runs diagonally across the box from D to F and is 18cm long.

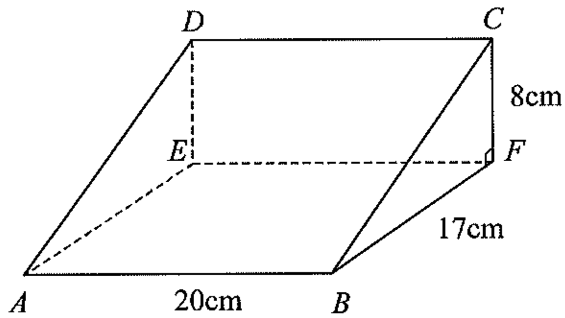
Calculate the length AE.
Give your answer correct to 3 significant figures.



3) The diagram shows a wedge in the shape of a prism.
Angle BFC is a right angle.

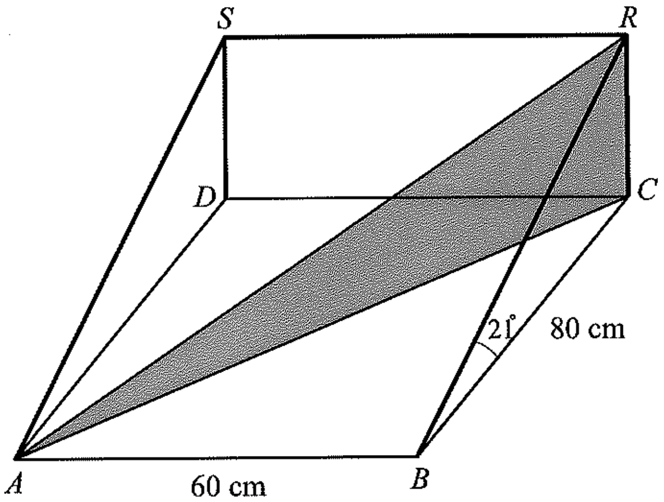
String runs diagonally across the wedge from A to C.

Calculate the length AC
Give your answer correct to 3 significant figures.



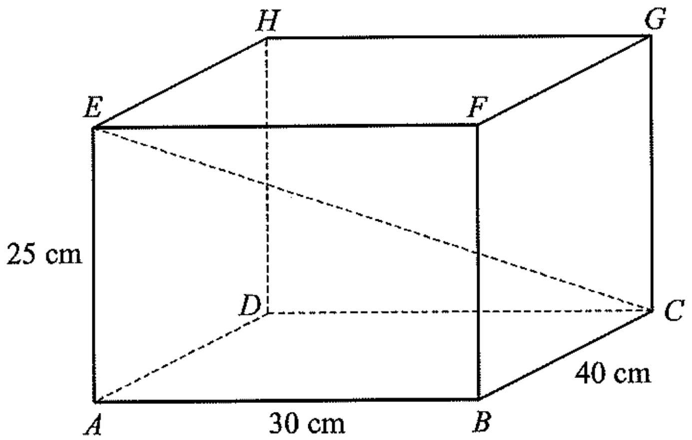
Trigonometry in 3D

1) The diagram shows a wedge.
The base of the wedge is a horizontal rectangle measuring 80 cm by 60 cm.
The sloping face *ABRS* makes an angle of 21° to the horizontal.



Calculate the angle that *AR* makes with the horizontal plane *ABCD*.
Give your answer correct to 1 decimal place.

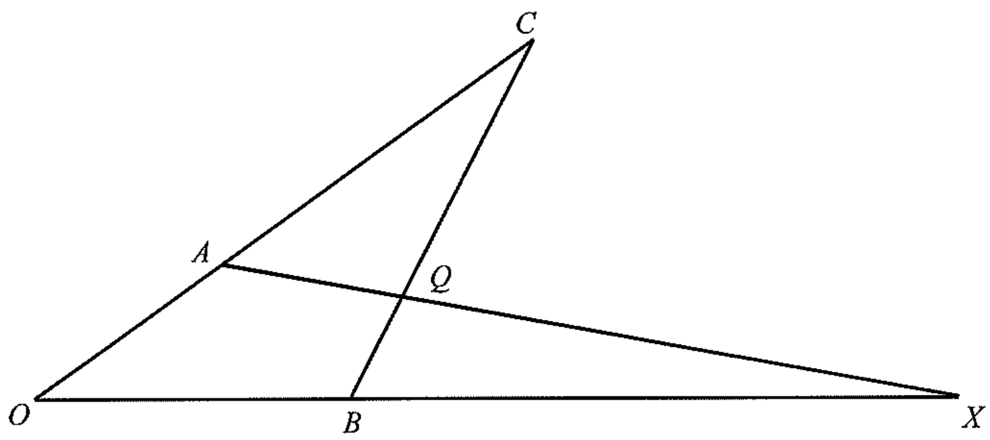
2) The diagram shows a box in the shape of a cuboid.
A string runs diagonally across the box from *C* to *E*.



a) Work out the length of the string *CE*.
Give your answer correct to 1 decimal place.

b) Work out the angle between the string *CE* and the horizontal plane *ABCD*.
Give your answer correct to 1 decimal place.

Vectors



In the diagram,

$\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OB} = 4\mathbf{b}$

OAC, *OBX* and *BQC* are all straight lines.

$AC = 2OA$ and $BQ : QC = 1 : 3$

a) Find, in terms of **a** and **b**, the vectors which represent

(i) $\overrightarrow{BC}$

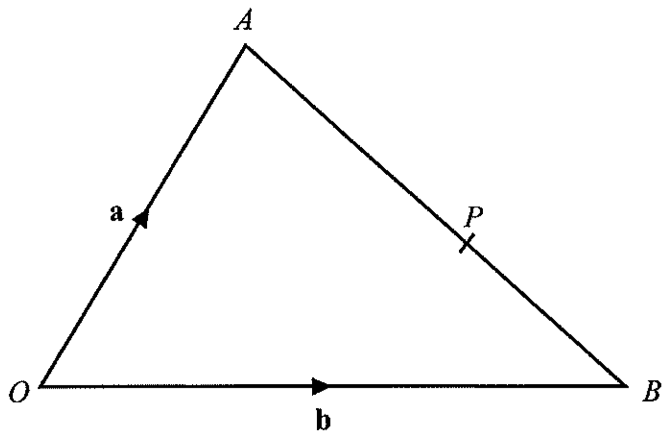
(ii) $\overrightarrow{AQ}$

Given that $\overrightarrow{BX} = 8\mathbf{b}$

b) Show that *AQX* is a straight line.

Vectors

1)



OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$

a) Find the vector *AB* in terms of **a** and **b**.

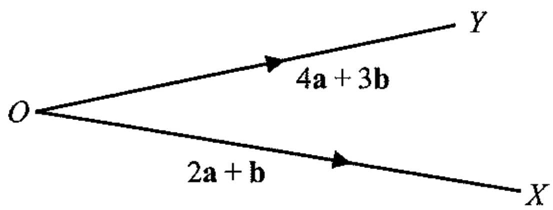
P is the point on *AB* so that $AP : PB = 3 : 2$

b) Show that $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

2)

$OX = 2\mathbf{a} + \mathbf{b}$

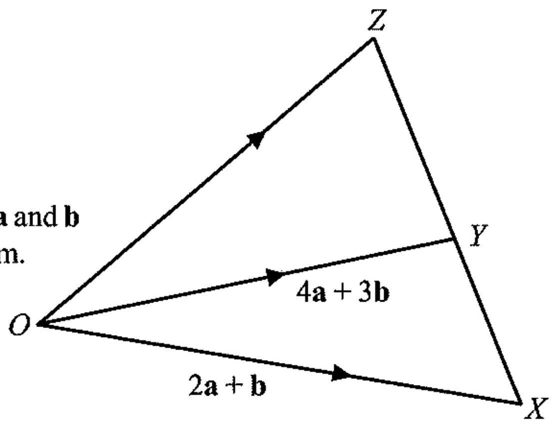
$OY = 4\mathbf{a} + 3\mathbf{b}$



a) Express the vector *XY* in terms of **a** and **b**
Give your answer in its simplest form.

XYZ is a straight line.
$XY : YZ = 2 : 3$

b) Express the vector *OZ* in terms of **a** and **b**
Give your answer in its simplest form.



Vectors

1) The diagram shows a trapezium *PQRS*.
$\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$.
PS is three times the length of *QR*.

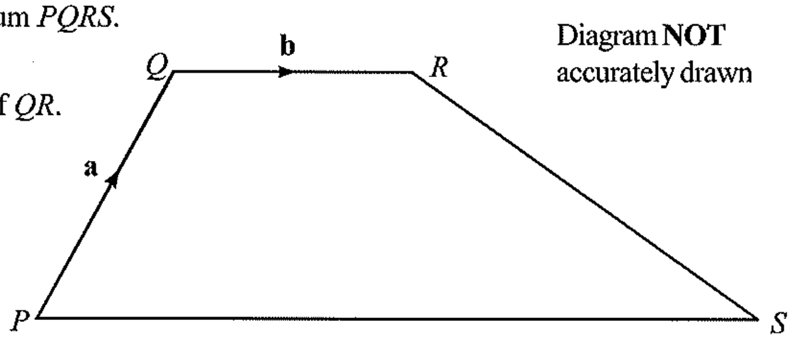


Diagram **NOT** accurately drawn

Find, in terms of **a** and **b**, expressions for
a) $\overrightarrow{QP}$ b) $\overrightarrow{PR}$ c) $\overrightarrow{PS}$ d) $\overrightarrow{QS}$

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2) In triangle *ABC*, *P* and *Q* are the midpoints of *AB* and *AC*.

$\overrightarrow{AP} = \mathbf{p}$ and $\overrightarrow{AQ} = \mathbf{q}$.

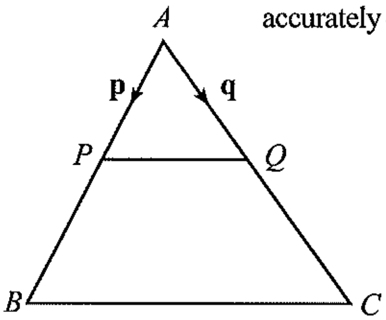

Diagram **NOT** accurately drawn

a) Find, in terms of **p** and **q**, expressions for
(i) $\overrightarrow{PQ}$ (ii) $\overrightarrow{AB}$ (iii) $\overrightarrow{AC}$ (iv) $\overrightarrow{BC}$

b) Use your results from (a) to prove that *PQ* is parallel to *BC*.

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3)

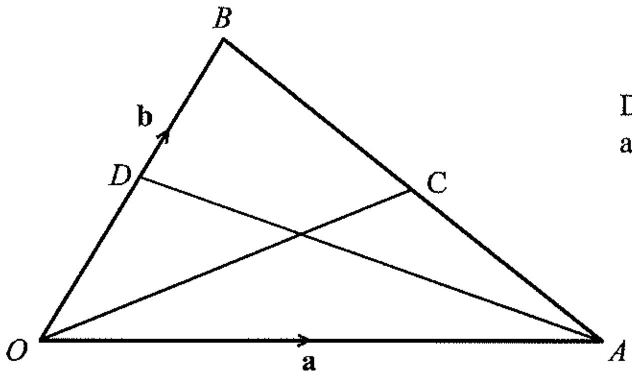

Diagram **NOT** accurately drawn

OAB is a triangle.
D is the midpoint of *OB*.
C is the midpoint of *AB*.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(i) Find $\overrightarrow{OC}$ in terms of **a** and **b**.

(ii) Show that *DC* is parallel to *OA*.

Vectors

1)

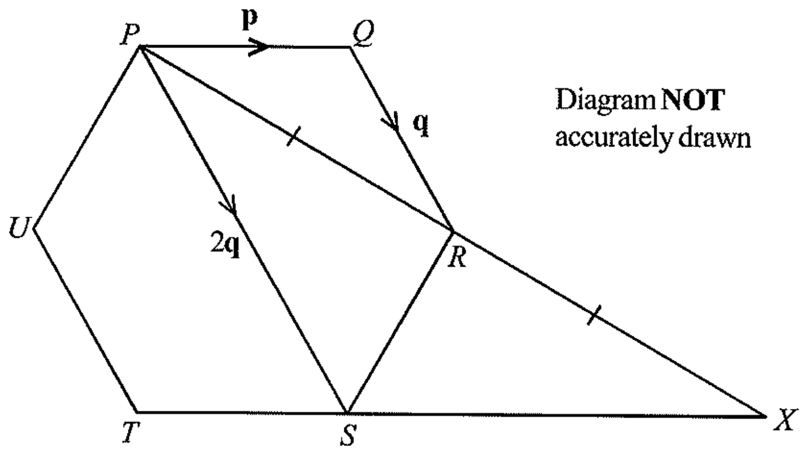

Diagram **NOT** accurately drawn

PQRSTU is a regular hexagon.

$\overrightarrow{PQ} = \mathbf{p}$ $\overrightarrow{QR} = \mathbf{q}$ $\overrightarrow{PS} = 2\mathbf{q}$

a) Find the vector *PR* in terms of **p** and **q**.

$\overrightarrow{PR} = \overrightarrow{RX}$

b) Prove that *PQ* is parallel to *SX*

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2)

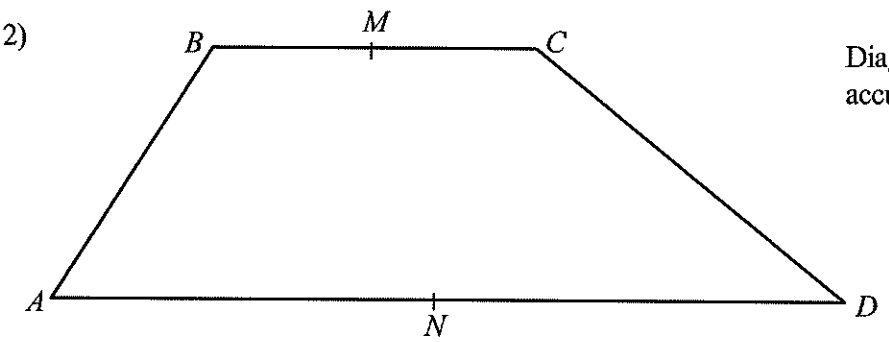

Diagram **NOT** accurately drawn

ABCD is a trapezium with *BC* parallel to *AD*.

$\overrightarrow{AB} = 3\mathbf{b}$ $\overrightarrow{BC} = 3\mathbf{a}$ $\overrightarrow{AD} = 9\mathbf{a}$

M is the midpoint of *BC* and *N* is the midpoint of *AD*.

a) Find the vector *MN* in terms of **a** and **b**.

X is the midpoint of *MN* and *Y* is the midpoint of *CD*.

b) Prove that *XY* is parallel to *AD*.